Journal of Pure Applied and Industrial Physics, *Vol.8(12),224-232, December 2018* *ISSN 2229-7596 (Print)*
(An International Research Journal), *www.physics-journal.org* *ISSN 2319-7617 (Online)*

Study of Abrikosor Vortex lattice and evaluation of $\left({K_1}/{K}\right)$ and $\left({K_2}/{K}\right)$ as a function of reduced temperature $\left({T}/{T_c}\right)_c$ for two N_b-T_1 superconductor

Shambhoo Nath Prabhakar

Department of Physics,
Mahila College, Tekari (Gaya), Vill : Andhar : P.O- Makhdumpur- 824232, INDIA.
email: shambhu91937@gmail.com.

(Received on: November 27, 2018)

ABSTRACT

Type- I Superconductor are superconductors that exhibit zero resistance and perfect diamagnetism. They are perfect diamegnets for applied magnetic fields below the critical field B_c and becomes normal for higher applied fields. Their coherence length exceeds their penetration depth ($\xi > \lambda$) so it is not energically favourable for boundaries to form between their normal and superconducting phase. The superconducting element with the exception of Niobium, are all type I.

When the penetration depth λ is larger than the coherence length ξ, it becomes energetically favorable for domain walls to form between the superconducting and normal regions. When such a superconductor, called type II, is in a magnetic field, the free energy can be lowered by causing domains of normal material containing trapped flux to form with low energy boundaries created between the normal core and the surrounding superconducting material. When the applied magnetic field exceeds a value referred to as the lower critical field, BC_1, magnetic field is able to penetrate in quantized units by forming cylindrically symmetrical domains called vortices. For applied fields slightly above BC1 the magnetic field inside a type II superconductor is strong in the normal cores of the vortices, decreases with distance from the cores, and becomes very small far away for much higher applied field the vortices overlap and the field inside the superconductor become strong everywhere. Eventually, when the applied field reaches a value called the upper critical field BC_2, the materials becomes normal. Alloys and compounds exhibit type II superconductivity with mixed type magnetic behavior and partial flux penetration above BC_1. Type II superconductors also have zero resistance, but their perfect diamagnetism occurs only below the lower critical field BC_1. Then one defines the ratio of λ and ξ as a Ginzburg-Landau parameter $_K$. $_K$ plays a very important role in type II superconductors. The density of super electron n_s which characterizes the superconducting state, increase from zero at the interface with a normal material to a constant value for inside, and the length scale for this to occur is the coherence length ξ. As external magnetic field B decays exponential to zero inside a superconductor. For type I superconductor for coherence length is the larger of the two length scales, so superconducting coherence is maintained over relatively large distance within the sample. The overall coherence of the superconducting electrons is not disturbed by the presence of external magnetic fields.

Keywords: Superconductivity, super current, vortex lattice, Gibbs energy.

INTRODUCTION

We have discussed in great detail about GL equation in chapter II. In fact one has given an account of calculation of second order critical fields with the use of limearized GL equations. We have $H_{C2}= K\sqrt{2}\ H_{cb}$. Within the framework of GL theory[1], k is independence. There is another field HC_3 where superconductivity appears in the form surface sheath with a with a thickness above coherence length $\xi\ (T)$ on surface parallel to the applied field. Our has $H_{c3} = 1.69\ k\sqrt{2}\ H_{cb} = 1.69H_{c2}$. Between H_{c2} and H_{C3} a super current can flow in the surface sheath, so that the resistance transition for a small measuring current occurs at H_{C3}. The magnetization on the other hand is governed by the behavior of the bulk of the specimen[2]. So that the magnetic transition occurs at H_{C2}. In type I superconductor which has $H_{C2}<H_{cb}<H_{c3}$ may be expected to appear as the critical supercooling field. It is also possible to have intermediate type superconductor with $H_{c2}<H_{cb}<H_{c3}$ in which the surface sheath appears in some field interval above H_{cb}.

Now at H_{C2} and H_{C3}. The order parameter ψ varies spatially, over the characteristics distance $\xi(T)$ whereas in calculating the critical field of a film of thickness d< $\xi(T)$, one is able to take Ψ as a constant. Now one goes to look at some consequences of the nonlinear term $\beta|\Psi|^2\ \Psi$ in the GL equation. In particular, one discusses the structure of the mixed state. Ψ various spatially and it is not possible to deal analytically with the nonlinear terms for all values of Ho. However, one is dealing with the second order phase transition, and just however, Hc_2. Ψ is small so that one can handle the nonlinear terms by a perturbation method. The solution of the problems is due to Abrilcosov[3]. In this chapter, we have discussed the Abrikosov vortex lattice and evaluated the ratio of $\left({K_1}/{K}\right)$ and $\left({K_2}/{K}\right)$ as a function of reduced temperature (T/T_c) for two Nb-Ti alloys. Our evaluated values of these two ratio decreases of (T/T_c). The value of K is taken from G.L. theory for the alloys. The result also indicate that the values are identical at T= Tc.

MATHEMATICAL FORMALUE USED IN THE EVALUATION

Now one uses first- Ginzburg-Landau equation which is given by

$$\frac{1}{2m} (- \text{it}\, \nabla - 2eA)^2\, \psi + \alpha\, \psi|\beta|^2\, \psi = 0 \qquad (1)$$

With the use of garge div A= 0. A is the vector potential. The expression for current density is given by

$$J_c = \frac{i\hbar e}{m} (\psi^* \nabla\psi - \psi\nabla\psi^X) - \frac{4e^2}{m} \psi^x\, \psi A \qquad (2)$$

This is the standard expression for a quantum mechanical current Here A enters only in the gradient terms and in the field energy terms $B^2/2\mu_0$. Here one uses the mass m rather than 2m which means only a change in the normalization of ψ. Equation (2) indicates that a superconductor is characterized by a macroscopic wave function. Equation (3) is a type of local expression.

We have to solve the two GL equation (3) for ψ and (2) for the current J_e. We take the applied field Ho to be just less than H_{c2}. So that the vector potential A satisfies.

$$\text{Curl } A = B = \mu_0 H_o + B_{loc} \qquad (3)$$

Where B_{loc} is the field generated by the super currents.

$$\text{Curl Bloc} = \mu_0 J_e \qquad (4)$$

It is convenient to write

$$A = A_{c2} + A_1 \qquad (5)$$

Where Ac2 is the vector potential of the field H_{c2}. Given by

$$A_{c2} = (0, = \mu_0 H_{c2} X, 0) \qquad (6)$$

We take all magnetic fields to be directed along the z axis.

Just below H_{c2}. We can expect ψ to be close to a solution of the linear equation (6), with H_{c2}, in place of H_0. We therefore put

$$\psi = \psi_L + \psi_1 \qquad (7)$$

Where ψ_L is solution of the linear equation. However, equation (7) begs tow questions about ψ_L is important because we have a nonlinear system, the normalization of ψ_L is important eventually the normalization determines the strength of the super current J_e and therefore the induction B. To make sure that we have the correct normalization, we shall require ψ_L and ψ_1 are orthogonal $\int \psi_L\, \psi_1\, d^3 r = 0$ (8)

We can expand ψ_1 as a series in terms of the eigenfunctions for the oscillator Hamiltonian, and equation (8) is the condition that the lowest eigenfunction ψ_L does not occur in ψ_1. The second equation about ψ_L arises because the lowest eigenstate is highly degenerate the solution can have any values of k_y and x_o. Again the nonlinearity of the system lifts the degeneracy, and singles out one particular solution. At the outset, however, we simply choose a general linear combination of solutions with various x_o values.

$$\psi_L (x,y) = \sum_n C_n \exp(\text{inky}) \exp[-(x-xn)^2 / 2\xi^2(T)] \qquad (9)$$

with

$$X_n = \text{nhk}/2e\mu_0 H_{c2} = \text{nk}\, \xi^2 (T) \qquad (10)$$

It will be seen that equation (9) is periodic in y with wavelength $2\pi/k$. Following the original calculation of Abrikosov, we shall consider only combination such that $|\psi_L|^2$ is periodic in x as well as in y. We can ensure this with the periodicity condition.

$$C_{n+N} = C_n \tag{11}$$

In fact the only value of N that have been considered are N=1 (Abrikosv 1957)[3] and N= 2 (Kleiner et al 1964)[4] the latter given the triangular lattice which is observed in practice. The choice of a periodic solution in (11) is nowadays rather obvious that the mixed state consists of a periodic array of vortices.

Most of the analysis is independent of the details structure of ψ_L that is the particular choice of C_n and k in equation (8). The first important result concerns the current J_L associated with ψ_L. If equation (9) is substituted into equation (2) for the current, it can be shown that

$$J_{Lx} = - \frac{eh}{m}\frac{\partial}{\partial y}|\psi_L|^2 \tag{12}$$

$$J_{Ly} = - \frac{eh}{m}\frac{\partial}{\partial x}|\psi_L|^2 \tag{13}$$

This means, by comparison with equation (4), that the field generated by J_L, which is the z direction of course is

$$B_{loc} = - \mu_0 e\hbar|\psi_L|^2 / m \tag{14}$$

This gives us the important result that the lines of constant B coincide with the lines of constant $|\psi L|^2$, and that these lines are also the lines of current flow, J_L. Thus the contours in diagrams like figure (A) and (B) represent at the same time level surface $|\psi L|^2$ and of B and streamlines of the current J_L.

We can now look at the normalization of ψ_L. We slipt us A and ψ as in equations (5) and (7), and rewrite the GL equation for ψ as an equation for the small correction ψ_1 :

$$\frac{2}{2m}(- i\hbar\nabla-2eA_{c2})^2 \psi_1 + \alpha \psi_1 + \frac{1}{2m} (- i\hbar\nabla - 2eA_{c2} - 2eA_1-)^2 \psi_L$$
$$+\alpha \psi_L+\beta|\psi_L|^2\psi_L = 0 \tag{15}$$

We treat, A_1, ψ_L and $\beta|\psi_L|^2$. As small quantities, and retain only first-order terms. For convenience, we have kept in the zeroth-order term $H_o\psi_L$, where

$$H_o= \frac{1}{2m} (- i\hbar\nabla - 2eA_{c2})^2 +\alpha \tag{16}$$

We have

$$H_o \psi_L = 0 \tag{17}$$

Since this is the defining equation for ψ_L. The term involving ψ_1 in equation (15) is $H_o \psi_1$. It we think again of ψ_1 as a sum of eigenfunctions of Ho, not including ψ_L. Hence our normalization condition, equation (8) is equivalent to

$$\int \psi^*_L H_o \psi_1 d^3r = 0 \tag{18}$$

We can substitute for $H_o \psi_1$ from equation (15), to get the explicit form

$$\int \left(\frac{1}{2m} \psi_L^* (-i\hbar\nabla - 2eA_{c2} - 2eA_1)\psi_L + \alpha|\psi_L|^2 + \beta|\psi_L|^4\right) d^3r = 0 \tag{19}$$

Equation (14) and (19) solve the problem for us, in principle, since equation (14) gives B in terms of $|\psi_L|^2$, and equation (19) determines $|\psi_L|^2$.

In order to make the solution explicit, we have to reorganize equation (19) somewhat. First, we integrate the first term by parts, to introduce $\nabla\psi_L^*$. The zeroth-order part of the first term cancels the integral of $\alpha|\psi_1|^2$, and the rest, ignoring the term in A_1^2, gives

$$\int(-A_1 .\ J_L + \beta|\psi|^4)\ d^3 r = 0$$

Where JL as before, is the current associated with ψ_L . With the use of equation (7) the first term here is

$$\int A_1 J_L d^3r = -\frac{1}{\mu_0}\int A_1\ culB_{loc} d^3r \quad (20)$$

$$= 1\frac{1}{\mu_0}\int B_{loc}.\ \text{Curl } A_1\ d^3 r \quad (21)$$

Where again we have integrated by parts, using the vector identity div (AxB) = B. Curl A-A. curl B. Form equation (3) and (5) we have.

$$\text{Curl}A_1 = \mu_0\ (H_o - H_{c2}) + B_{loc} \quad (22)$$

Putting equation (22) and (21) into equation (20) and using equation (14) for B_{loc}, we find

$$(2k^2 - 1)\frac{eh}{m} < |\psi_L|^4 > = (H_{c2} - H_o) < |\psi_L|^2 > \quad (23)$$

Where we have introduced the notation

$$\int |\psi_L|^2\ d^3r = \langle\psi_l\rangle^2 \qquad \text{etc.} \quad (24)$$

and replaced β by K^2

Equation (23) essentially as far as we can go without investigating the detailed form of the flux lattice described by ψ_L. It is convenient to summaries the properties of the lattice, following Abrikosov, in the parameter β_A :

$$\beta_A = \langle|\psi_L|^4\rangle / (\langle|\psi_L|^2\rangle) \quad (25)$$

Clearly we have $\beta \geq 1$ for any form of ψ_L. Equation (23) then yield

$$\langle|\psi_L|^2\rangle = \frac{m}{eh}\frac{H_{c2} - H_0}{(2K^2 - 1)\beta_A} \quad (26)$$

The average induction, from equation (3) and (14) is

$$\langle B\rangle = \mu_0 H_0 - \frac{\mu_0 H_{c2} - H_0}{(2K^2 - 1)\beta_A} \quad (27)$$

and equivalently the magnetization is

$$M = \frac{\mu_0 H_{c2} - H_0}{(2K^2 - 1)\beta_A} \quad (28)$$

It is easy to see from equation (28) that the Gibbs energy G decreases as β_A decreases. The choice of the parameters C_n and k in equation (9) must therefore be made in such a way as to minimize β_A. Abrikosvo originally chose all C_n equal and $k = (2\pi)^{1/2} / \xi(t)$, which gives a square lattice, illustrated in figure 3A with $\beta_A = 1.18$. Later, Klenier et al (1964)[1] showed that the choice N=2 in the periodicity equation (11) together with

$$C_1 = \pm\ iC_o \quad (29)$$

Which corresponds to a triangular lattices give $B_A = 1.16$. Furthermore, the square lattice can be sheared continuously into the triangular lattice with B_A decreasing all the time, so that there is no equation of metastability of the square lattice. The triangular lattice is shown in figure 3B will be seen that it is the same as the lattice photographed by Essmann and Trauble (1967)[5].

Perhaps the most important features of the results we have just derived is that the magnetization, equation (28) depends upon the same parameters K as gives the critical field H_{c2}. The GL equation, in the form in which we have stated them, are obviously valid only in some temperature interval near T_c. Since we started with Landau's assumption that the free energy can be expanded as a power series in the order parameter. In fact, for alloys, in which the electrodynamics are local, the microscopic theory can be solved at all temperatures of H ~ H_{c2}, using a method based on the Abrikosov calculation. The result is that the equation for H_{c2}. And the magnetization M continue to hold, except that the K parameter involved are different function of temperature.

$$H_{c2} = K_1 (T) \sqrt{2}\, H_{cb} \tag{30}$$

$$M = -\frac{\mu_0(H_{c2} - H_0)}{\left[2K\frac{2}{2}(T) - 1\right]\beta_A} \tag{31}$$

From this more recent point of view, one can say that the result of the GL calculation is that the K parameters coincide at T = T_c.

DISCUSSION OF RESULTS

In this paper, we have studied the Abrikosov vortex lattice and evaluated the ratio of $\left({}^{k_1}/_{k}\right)$ and $\left({}^{k_2}/_{k}\right)$ for two Nb-T_i alloys smaple superconductor. We have taken the value of k from Gizburg. Landau values. The evaluation has been performed as a function of reduced temperature $\left({}^{T}/_{T_c}\right)$. The results are shown in table $3T_1$ and $3T_2$ respectively for the given two samples. Our theoretical results indicated the ratio $\left({}^{k_1}/_{k}\right)$ and $\left({}^{k_2}/_{k}\right)$ both reduces as a function of $\left({}^{T}/_{T_c}\right)$. Coincides at T= T_c. The limiting values of T_c is the same for both. These results are consistent with the theoretical results of other workers [6-15]

Table: 1

Evaluation of $\left({}^{k_1}/_{k}\right)$ and $\left({}^{k_2}/_{k}\right)$ as function of reduced temperature $\left({}^{T}/_{T_c}\right)$for Nb-Ti alloy 37% (sample I) superconductor k = 0.84 is the GL Value

$\left({}^{T}/_{T_c}\right)$	$\left({}^{k_1}/_{k}\right)$	$\left({}^{k_2}/_{k}\right)$
0	1.55	2.50
0.1	1.50	2.40
0.2	1.46	2.10
0.3	1.42	2.00
0.4	1.40	1.89
0.5	1.38	1.80
0.6	1.36	1.70
0.7	1.36	1.70
0.8	1.32	1.50
0.9	1.30	1.40
1.0	1.25	1.30

Table: 2

Evaluation of $\left({}^{k_1}/_{k}\right)$ and $\left({}^{k_2}/_{k}\right)$ as function of reduced temperature $\left({}^{T}/_{T_c}\right)$ for Nb-Ti alloy 43% (sample II) superconductor k = 0.84 is the GL Value

$\left({}^{T}/_{T_c}\right)$	$\left({}^{k_1}/_{k}\right)$	$\left({}^{k_2}/_{k}\right)$
0	1.40	2.55
0.1	1.32	2.50
0.2	1.28	2.45
0.3	1.26	2.37
0.4	1.22	2.33
0.5	1.20	2.30
0.6	1.18	2.27
0.7	1.16	2.23
0.8	1.14	2.20
0.9	1.13	2.18
1.0	1.12	2.16

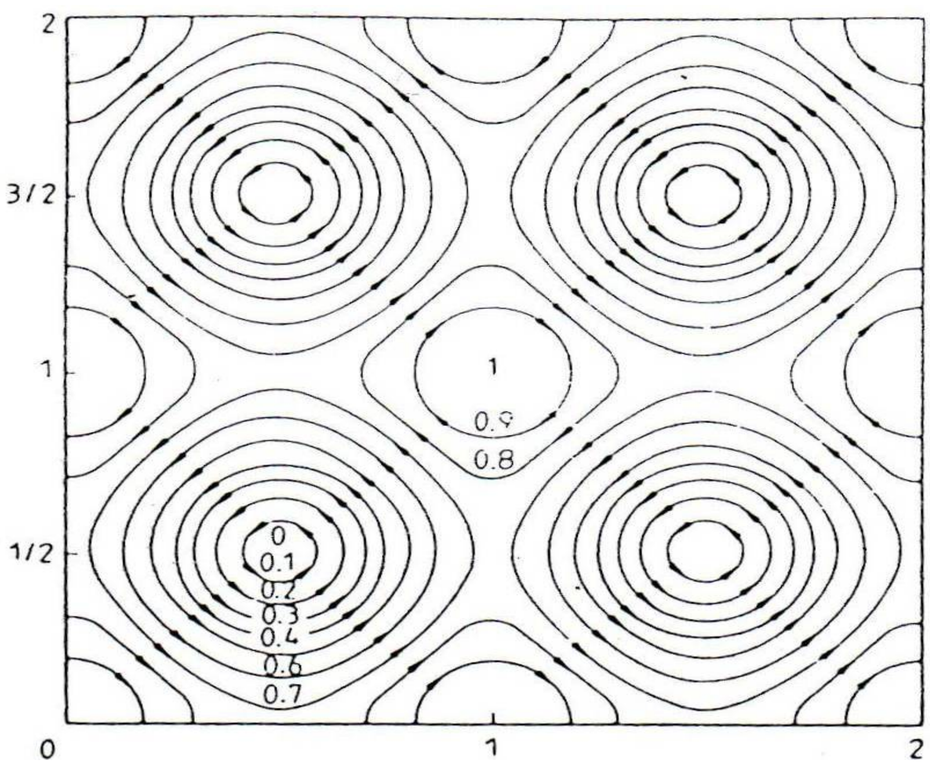

Figure: 1 Level surfaces of $|\psi_L|^2$ for the Abrikosov square lattice. The axes are marked in units of $(2\pi)^{1/2}$ ξ(T). (From Abrikosov 1957)

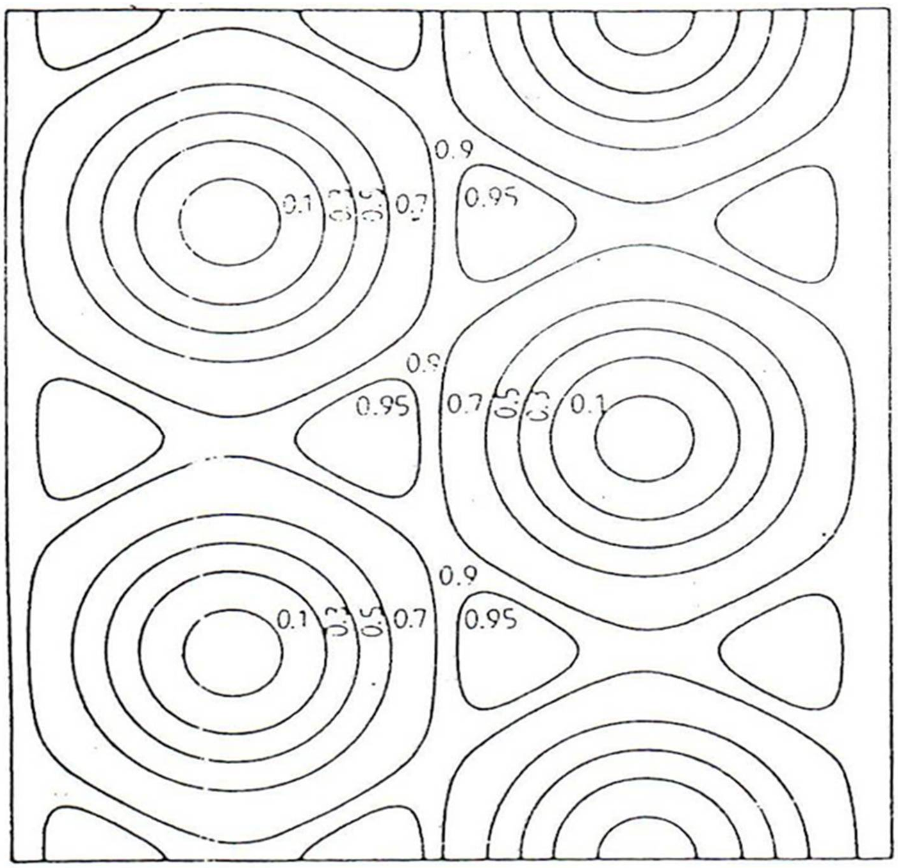

Figure 2 : Level surfaces of $|\psi_L|^2$ for the triangular lattice. The vertical distance between vortex cores is $2\pi^{1/2}\xi(T)/3^{1/4}$ and the horizontal distance between the rows of vortices is $3^{1/4}\,\pi^{1/2}\,\xi\,(T)$. (From Kleiner et al 1964).

CONCLUSION

As a from problem we have studied the Abrikosov we have studied the Abrikosov vortex lattice and evaluated the ratio of $\left({k_1}/{k}\right)$ and $\left({k_2}/{k}\right)$ for two Nb-T_i alloys sample superconductor. We have taken the value of k from Gizburg. Landau values. The evaluation has been performed as a function of reduced temperature $\left({T}/{T_c}\right)$. The results are shown in table 1 and 2 respectively for the given samples. Our theoretical results indicates that the ratio $\left({k_1}/{k}\right)$ and $\left({k_2}/{k}\right)$ both reduces as a function of $\left({T}/{T_c}\right)$, The decrease is more pronounced for higher value of $\left({T}/{T_c}\right)$, Coincides at $T=T_c$. The limiting values of T_c is the same for both. These results are consistent with the theoretical results of other workers[6-15].

REFERENCES

1. V.L. Ginzburg and L.D. Landau, *Zh. Eksp. Teor. Fiz* 20, 1064 (1980).
2. I.I. Geguizin, I. Ya Nikitoror and G.I. Alperoviteh, *Fiz. Tvend Tela* 15, 931 (1973).
3. A. A. Abrikosov, *Sov. Phys. JETP* 5 1174 (1957).
4. W.H. Kleiner, L.M. Rott and S. H. Autler, *Phys. Rev*. 133A 1226 (1964).
5. U. Essmann and H. Trauble, *Phys. Lett.* 24A, 526 (1967).
6. D. Goldschmidh *Phys Rev B*39. 2372 (1989).
7. A. Gold and A. Ghazali, *Phys Rev. B*43m 12952 (1991).
8. J.B. Goodenough, HJ.S. Zhou and J. Chan, *Phys. Rev B* 47. 5275 (1993).
9. N. H. Huer, N.H. Kim, S.H. Kim, Y.K. Park and J.C. Park, *Physica C* 231 227 (1994).
10. B.I. Ivlev and R.S. Thompson, *Phys. Rev. B* 57, 875 (1995).
11. Z. Iqbal, *Supercond. Rev*. 5, 49, (1996).
12. F. Irie and K. Yamtuji, *J. Phys. Soc. Jpn*. 63, 255 (1996).
13. K.P. Jain and D.K. Ray, *Phys. Rev*. 55, 12322 (1996).
14. S. Kivebon. *Physica C* 234, 567 (1996).
15. A. Khurana, *Phys. Rev. B* 60, 4316 (1997).