An Evaluation of Total Flux of the Vortex Present in the Core \( \frac{\phi_{\text{core}}}{\phi_0} \) as a Function of GL Parameter K

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(Received on: September 6, 2018)

ABSTRACT

We have evaluated the total flux of the vortex present in the core \( \frac{\phi_{\text{core}}}{\phi_0} \) as a function of Ginzburg Landau parameter K. The evaluation has been done under in two approximaton. Our in the shown under high \( \kappa \) approximation and other due to Tinkham. For high \( \kappa \) approximations the conditions is that effective penetration depth \( -\lambda \) should be greater than effective coherence length \( \xi \). This is valid for high temperature superconductor which are type II superconductor having GL paramet k= 100. The vortex is assumed to be infinitely long and axially symmetric so that there are no z or angular dependences of its field distributions. The problems this equipment to two dimensional problem of determining the radial dependences. We have computed the value of \( \frac{\phi_{\text{core}}}{\phi_0} \) as a function of \( k \) we have compared our result with those obtained from Tinkham. Our results indicate that \( \frac{\phi_{\text{core}}}{\phi_0} \) decreases very fastly with the value of \( \kappa \). Similar observation has also been noticed from Tinkham formalism under high \( \kappa \) approximation.

Keywords: Superconductors, Vortices, Landon formation, Bessel function, Euler Mascheroni Constant.

INTRODUCTION

Type I superconductors are perfect diamagnets for applied magnetic fields below the critical field \( B_c \) and become normal in higher applied fields. Their coherence length exceeds their penetration depth so its is not energetically favourable for boundaries to form between their normal and superconducting phases. On the other hand in case of type II superconductor \( \lambda > \xi \) and it becomes energetically favourable for domain walls to form between superconducting and normal regions. When type II superconductors are placed in a magnetic
field and if the applied magnetic field $B_{app}$ exceeds a value $B_{c1}$ (lower critical field) the magnetic flux is able to penetrate in quantized units forming cylindrically symmetric domains called vortices. For $B_{app}$ slightly above $B_{c1}$ the magnetic field inside a type II superconductor is strong in the normal cores of the vortices, decreases with distance from the cores and becomes very small for away. For much higher applied fields the vortices overlap and field inside superconductor becomes strong everywhere. Eventually, when the applied field reaches a value called the upper critical field ($B_{c2}$), the material becomes normal.

MATHEMATICAL FORMULAE USED IN THE EVALUATION

(A) Critical Fields

We know that type I superconductor has a critical field $B_c$ given by

$$G_n - G_s = \frac{B_c^2}{2\mu_0}$$  \hspace{1cm} (1)

Where $G_n$ and $G_s$ are the Gibbs free energy of normal and superconducting state $\frac{B_c^2}{2\mu_0}$ is the magnetic energy of this critical fields. Equation (1) is thermodynamic expression, $B_c$ is called the thermodynamic critical fields. Both type I and type II superconductors have thermodynamic critical fields. In addition, a type II superconductor has lower critical field to the quantum flux $\phi_0$, Ginzburg-Landau (G-L) parameter $K$ and penetration depth $\lambda$ and coherence length $\xi$ by the following expressions.$^{3-7}$

$$B_{c1} = \frac{\phi_0 ln k}{4\pi\lambda^2}$$  \hspace{1cm} (2)

$$B_{c2} = \frac{\phi_0}{2\pi \xi^2}$$  \hspace{1cm} (3)

These can be expressed in terms of the thermodynamic critical field $B_c$.

$$B_c = \frac{\phi_0}{2\sqrt{2\pi k\lambda}}$$  \hspace{1cm} (4)

Using (4) equation (2) and (3) assumes the form

$$B_{c1} = \frac{B_c ln k}{\sqrt{2 K}}$$  \hspace{1cm} (5)

$$B_{c2} = \sqrt{2 K B_c}$$  \hspace{1cm} (6)

The ratio and product of two critical fields are given by

$$\frac{B_{c2}}{B_{c1}} = 2k^2 / ln k$$  \hspace{1cm} (7)

$$(B_{c1} B_{c2})^{1/2} = B_c (ln k)^{1/2}$$  \hspace{1cm} (8)

When the applied magnetic field is perpendicular to the surface of the superconductor, the upper critical is truly $B_{c2}$. When it is parallel to the surface it turns out$^{6,7}$ one that the superconducting state can persist in a thin surface sheath for applied surface field up to the higher values $B_{c3} = 1.69 B_{c2}$. 

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(B) Vortices

As one knows that an applied magnetic field $B_{\text{app}}$ penetrates a superconductor in the mixed state, $B_{c1} < B_{\text{app}} < B_{c2}$. Penetration occurs in the form of tubes called vortices which are to confine the flux.\(^8\) The highest field in the core has a radius $\xi$. Within which magnetic flux and screening currents flowing around the core are present together. The current density $J_s$ of these shielding current decays with distance from the core in an approximately exponential manner. Analytical expressions for the distance dependence of $B$ and $J$ are desired in the high-kappa ($k \rightarrow 1$) approximation.

(C) High-Kappa Approximation

To obtain a description of vortices in a more quantitative nature, one obtains an expression for the distance dependence of the confined magnetic fields. For the high-K limit, $\lambda >> \xi$ which is valid for the copper oxide superconductor, one can use Helmholtz equation that where derived from London formation. The vortex is assumed to be infinitely long and axially symmetric so that there are no $z$ or angular dependences of its field distribution. The problem is true equivalent to the two dimensional problem of determining the radial dependences.

The magnetic field of the vortex is in the $z$ direction and its radial dependence outside the vortex core is obtained from Helmholtz equation given by in Cartesian coordinate.

$$\nabla^2 B = \frac{\vec{B}}{\lambda_L},$$

$\lambda_L$ = London penetration depth.

Here, one writes the Helmholtz equation in cylindrical coordinate for the two dimensional case of axial symmetry without assuming any angular dependence.

$$\lambda^2 \frac{d}{dr} \left( r \frac{d}{dr} \right) B - B = 0 (\lambda_L = \lambda)$$

This equation has an exact solution.

$$B(r) = \frac{\phi_0}{2\pi \lambda^2} k_0 \left( \frac{r}{\lambda} \right)$$

Where $K_0 \left( \frac{r}{\lambda} \right)$ is a zeroth-order modified Bessel function. Using equation (2) this can be

$$B(r) = B_{c1} \frac{k_0 \left( \frac{r}{\lambda} \right)}{\frac{1}{2} n k}$$

To obtain the current density one substitutes equation (11) in the Maxwell equation for $B_m$ (magnetic field inside superconductor)

$$\nabla \times B_m = \mu_0 J_s$$

From equation (4.13) one obtains

$$J_s (r) = \frac{\phi_0}{2\pi \mu_0 \lambda^2} K_1 \left( \frac{r}{\lambda} \right)$$

$$J_{s1} \frac{k_1 \left( \frac{r}{\lambda} \right)}{\frac{1}{2} n (k)}$$
Where $K_1(r/\lambda)$ is first-order modified Bessel function and the characteristic current density $J_{c1}$ is defined in analysis of relation.

$B_c = \frac{\mu_0 J_c}{\mu_0 \lambda}$

$J_{c1} = \frac{B_c}{\mu_0 \lambda}$ (15)

The function $K_1(r/\lambda)$ results from differentiation of equation (13) as one obtains the modified Bessel function recursion relation.\\

\[k_1(X) = d K_0(X) / dx\] (16)

The current density also satisfies the Helmholtz equation expressed in cylindrical coordinates.

\[\frac{\lambda^2}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) J_s + J_s = 0 \] (17)

The distance dependence of the modified Bessel functions and $K_0(r/\lambda)$ and $K_1(r/\lambda)$ are associated with $B(r)$ and $J_s(r)$ respectively. These modifed Bessel functions have anymptotic behaviours at small radial distance.\\

\[K_0 \left( \frac{r}{\lambda} \right) \approx \ln \left( \frac{2\lambda}{r} \right) - r \quad r << \lambda \] (18a)

\[K_1 \left( \frac{r}{\lambda} \right) \approx r / r \quad r << \lambda \] (19)

\[K_0 \left( \frac{r}{\lambda} \right) \approx \exp \left( \frac{-r/\lambda}{(2r/\pi \lambda)^{1/2}} \right) \quad r >> \lambda \] (19a)

\[K_1 \left( \frac{r}{\lambda} \right) \approx \exp \left( \frac{-r/\lambda}{(2r/\pi \lambda)^{1/2}} \right) \quad r >> \lambda \] (19b)

These large distance expressions permits ones to express the magnetic field and current density for from the core in the form

\[B \approx B_{c1} \frac{(2\pi)^{1/2}}{\ln(k)} \exp \left( -\frac{r/\lambda}{(r/\lambda)^{1/2}} \right) \quad r >> \lambda \] (20a)

\[J_s \approx J_{c1} \frac{(2\pi)^{1/2}}{\ln(k)} \exp \left( -\frac{r/\lambda}{(r/\lambda)^{1/2}} \right) \quad r >> \lambda \] (20b)

One sees from equation. (18a) and (18b) that both $B$ and $J_s$ are singular at $r=0$. Since the core is so small in the high kappa approximation, it is appropriate to remove the singularity by assuming that the magnetic field in the core is constant with the value $B(0)$ given in equation (11) for $r = \xi$. Even if the mathematical singularity were not removed the total flux would still remain finite at $r\to0$.\n
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Using equation (11), (12), (18) (a) one obtains the expression for the fraction of the total flux in the vortex that is present in the core.

\[ \Phi_{\text{core}} \equiv \left( \frac{\Phi_0}{2k^2} \right) \left[ 1_{\pi \left( 2k \right)} + \frac{1}{2} - 1_\gamma \right] \]  

(21)

We have computed the values of \( \Phi_{\text{core}} / \Phi_0 \) as a function of k which is shown in table 1.

Tinkham\textsuperscript{11} used high-\( K \)-approximation by writing the \( K_0 \left( \frac{r}{\lambda} \right) \) in the following form.

\[
B(r) = \frac{\Phi_0}{2\pi\lambda^2} \left[ \frac{\pi \lambda}{2 \, r} \right]^{1/2} e^{-r/\lambda} \quad r \to \infty
\]  

(22a)

\[
B(r) \cong \frac{\Phi_0}{2\pi\lambda^2} \left[ 1_{n \lambda / r} + 0.12 \right] \quad \xi < r < \lambda
\]  

(22b)

\[
\Phi_{\text{core}} \cong \frac{\Phi_0}{\sqrt{2\pi k^{5/2}}} \left[ 1_{n k} + 0.12 \right]
\]  

(23)

We have also computed equation (23) as a function of k and results an also shown in the table 1.

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<th>Tinkham</th>
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**DISCUSSION OF RESULTS**

In this chapter, we have evaluated the total flux of the vortex present in the core \( \Phi_{\text{core}} / \Phi_0 \) as a function of Ginzburg Landau parameter K. The evaluation has been done under in two approximations. Our in shown under high \( K \)-approximation and other due to Tinkham.\textsuperscript{11} For high \( K \) approximations the conditions\textsuperscript{12-15} is that effective penetration depth – \( \lambda \) should be greater than effective coherence length \( \xi \). This is valid for high temperature
superconductor which are type II superconductor having GL paramet k=100. The vortex is assumed to be infinitely long and aximallysymmetic so that there are no z or angular dependences of its field distributions. The problems this equivalent to two dimensional problem of determining the radial dependences. We have computed the value of $\Phi_{\text{core}}$ as a function of k using equation (21), we have compared our result with those obtained from Tinkham who has written $k_0 \left( \frac{V}{\lambda} \right)$ in the form shown in equation (22a) and (22b). Our results indicate that $\left[ \frac{\Phi_{\text{core}}}{\Phi_0} \right]$ decreases very fastly with the value of k. Similar observation has also been noticed from Tinkham formalism under high $\kappa$ approximation.

CONCLUSION

From the above investigations on type II superconductivity we can draw the following conclusion.
1. Ginzburg-Landau phenomenological theory works quite well in explaining the various properties of type II superconductors.
2. Tinkham high-Kappa approximation works very well in the evaluation of flux in the vortex core.

REFERENCES