

# A Theoretical Study of Spin-orbit Coupling in Polarized Fermionic Condensate

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## ABSTRACT

Using the theoretical formalism of R Liao *et al.*<sup>9</sup> and Y Zhang *et al.*<sup>10</sup>, we have theoretically studied spin-orbit coupled fermionic condensate. We have evaluated various parameters as a function of SOC parameter. The evaluated parameters are tricritical temperature, tricritical magnetic field, tricritical polarization and two types of critical temperature  $T_{CO}$  and  $T_{Cg}$  all as a function of SOC parameter. Our theoretically obtained results show that all parameter increase and decrease with SOC parameter.

These results indicate that there is a competition between SOC and population imbalance due to which these new features have been observed. We hope that the current work will add new excitements in the field of cold atom physics involving artificial gauge field and spin orbit coupling.

**Keywords:** Fermionic condensate, Spin-orbit coupling, Population imbalance, Finite temperature phase diagram, Topological insulator, Spin Hall effect, Rashba spin-orbit term, SOC parameter

## INTRODUCTION

The recent realization of synthetic gauge field<sup>1</sup> and spin-orbit coupling<sup>2</sup> opens a new arena in the field of quantum many-body system. This helps to explore the physics of ultra cold atoms. The spin orbit coupling (SOC) with equal Rashba and Dresselhaus strength in a

neutral Bose-Einstein condensate were studied by dressing two atomic spin states with a pair of lasers. Due to SOC, one observed spin Hall effect and topological insulator. This has opened a new and very exciting field in condensed matter community. The beauty with SOC is that it is applicable to both bosons and fermions. This freedom has allowed one to discover topological insulators and topologically nontrivial states in fermionic neutral atom system<sup>3,4</sup> Now, one tries to study SOC in fermionic atoms as <sup>6</sup>Li and <sup>40</sup>K. The much attention has been paid to know the physics of BEC condensate, BEC crossover and polarized Fermi gas in the presence of SOC. The SOC has been predicted to lead to various new phenomena like two-body bound state even in BCS side ( $a_s < 0$  of a resonance. For the many body physics at mean field level, it enhances BCS pairing via the increased density of states at low energy. It leads to anisotropic super fluids through mixing of spin singlet and triplet component.

Polarized fermionic condensates have been focused both theoretical and experimental research over the past one decade<sup>5</sup>. Various possible asymmetric super fluid phases have been proposed like anisotropic or in homogeneous . super fluid state with crystalline structure popularly known as FFLO(Flude-Ferrel-Larkin-Ovchinnikov) deformed Fermi surface, a homogeneous gapless super fluid state (breached pair), phase separation into the normal Fermi gases and fully paired super fluid atate.<sup>6</sup> Now a question will arise asto how SOC reshapes our understanding of these exciting systems. Most of the theoretical studies<sup>7,8</sup> have focused mainly on zero temperature leaving the physics at finite temperature, which is experimentally relevant, largely intact.

## MATHEMATICAL FORMULA USED IN THE STUDY

One considers three-dimensional homogeneous two species polarized Fermi gases interacting via an attractive contact potential. The Hamiltonian also consists of an isotropic Rashba spin orbit coupling. The Hamiltonian is written in the following form:

$$H = \int d^3r \rightarrow \sum_{\sigma=\uparrow,\downarrow} \psi_{\sigma}^{\dagger}(r \rightarrow) \left[ \frac{p^2}{2m} - \mu_{\sigma} \right] \psi_{\sigma}(r \rightarrow) - g \int d^3r \rightarrow \psi_{\uparrow}^{\dagger}(r \rightarrow) \psi_{\downarrow}(r \rightarrow) \psi_{\downarrow}(r) \psi_{\uparrow}(r \rightarrow) + H_{so} \quad (1)$$

$$H_{so} = \lambda \sum_{\mathbf{k}} k_{\perp} [e^{-i\varphi_{\mathbf{k}}} \psi_{\mathbf{k}\uparrow}^{\dagger} \psi_{\mathbf{k}\downarrow} + H.c] \quad (2)$$

Here,  $k_{\perp} = (k_x, k_y)$ ,  $\varphi_{\mathbf{k}} = \text{Arg}(k_x + k_y)$ . The strength of spin-orbit coupling  $\lambda$  can be tuned by atom-laser interaction. One defines the chemical potential  $\mu$  and the magnetic field  $h$  such that  $\mu_{\uparrow} = \mu + h$  and  $\mu_{\downarrow} = \mu - h$ . The spin imbalance between two species is denoted by the

polarization  $P = \frac{(n_{\uparrow} - n_{\downarrow})}{(n_{\uparrow} + n_{\downarrow})}$ . The interaction strength  $g$  is represented in terms of s-wave

scattering length  $a_s$  by using the prescription

$$\frac{m}{4\pi a_s} = -\frac{1}{\pi} + \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{2\varepsilon_{\mathbf{k}}} \quad (3)$$

$$\varepsilon_k = \frac{k^2}{2m} \quad (4)$$

$$(\hbar = k_B = 1)$$

## DISCUSSION OF RESULTS

Using the theoretical formalism of R Liao<sup>9</sup> *et al.* and Y Zhang *et al.*<sup>10</sup>, we have studied the spin-orbit coupled fermionic condensate. In this formalism, one investigated a two component atomic Fermi gas with population imbalance in the presence of Rashba type spin-orbit coupling. As a competition between SOC and population imbalance the finite temperature phase diagram reveals a large varieties of new features. This includes the expanding of super fluid state regime and the shrinking of both the phase separation and the normal regimes. It was observed that for large value of SOC parameter the phase separation disappears. It gives way to the super fluid state. One also observes that the transition point moves towards a regime of low temperature, high magnetic field and high polarization as the SOC increases. In Table

T1, we have shown the evaluated results of  $\frac{T_{tr}}{T_F}$  as a function of  $\frac{\lambda}{v_F}$  for polarized fermionic condensate. Here,  $T_{tr}$  is the tricritical point, and  $T_F$  is the Fermi temperature.  $\lambda$  is SOC parameter and  $v_F$  is the Fermi velocity. Our theoretically evaluated results show that  $\frac{T_{tr}}{T_F}$

decrease with  $\frac{\lambda}{v_F}$ . In table T2., we have evaluated the results of the ratio  $\frac{h_{tr}}{E_F}$  as a function of SOC parameter. Here  $h_{tr}$  is the critical magnetic field. Our theoretically evaluated results indicate that the ratio of tricritical magnetic field increases with the SOC parameter. Here  $E_F$  is the Fermi energy. In table T3, we have shown the evaluated results tricritical polarization  $P_{tr}$  as a function of SOC parameter. Our theoretically obtained results indicate that tricritical polarization  $P_{tr}$  also increases with SOC parameter. In table T4, we have shown the evaluated results of Polarization  $P$  as a function of magnetic field with various values of SOC parameter. Our theoretically obtained results show that Polarization increase with magnetic field. For each value of SOC parameter. The increase is large for low value and small for large value of SOV parameter. In table T5, we have evaluated critical temperature for balanced super fluid  $T_{CO}$  and  $T_{Cg}$  as a function of SOC parameter. All these temperature are calculated at Unitarity limit.  $T_{CO}$  is evaluated by mean field theory and  $T_{Cg}$  Using N-S-R theory. Our theoretically evaluated results show that  $T_{CO}$  has almost constant value for SOC parameter and  $T_{Cg}$  increases very fastly. Our theoretically evaluated results are in good agreement other theoretical workers<sup>11-15</sup>.

**Table T1**

**An evaluated result of ratio of Tricritical temperature to Fermi temperature ( $T_{tr}/T_F$ ) as a function of SOC strength  $\lambda$  to Fermi velocity  $v_F$  ( $\frac{\lambda}{v_F}$ ) for polarized fermionic condensate**

$\frac{\lambda}{v_F}$	$\frac{T_{tr}}{T_F}$
0.0	0.083
0.01	0.075
0.015	0.072
0.02	0.067
0.025	0.058
0.03	0.049
0.035	0.042
0.04	0.038
0.045	0.032
0.05	0.027
0.055	0.022
0.06	0.015

**Table T2**

**An evaluated result of tricritical point  $\frac{h_{tr}}{E_F}$  as a function of SOC parameter  $\frac{\lambda}{v_F}$  for polarized fermionic condensate**

$\frac{\lambda}{v_F}$	$\frac{h_{tr}}{E_F}$
0.00	0.125
0.010	0.132
0.015	0.140
0.020	0.145
0.025	0.152
0.030	0.154
0.035	0.163
0.040	0.169
0.045	0.172
0.050	0.174
0.055	0.178
0.060	0.182

**Table T3**

An evaluated result of polarization  $P_{tr}$  as a function of SOC strength  $\frac{\lambda}{v_F}$  for polarized fermionic condensate

$\frac{\lambda}{v_F}$	$P_{tr}$
0.01	0.153
0.015	0.187
0.02	0.224
0.025	0.256
0.03	0.297
0.035	0.325
0.04	0.376
0.045	0.418
0.05	0.475
0.055	0.489
0.06	0.532
0.065	0.554

**Table T4**

An evaluated result of polarization P as a function of magnetic field  $\frac{h}{E_F}$  for different value of SOC parameter  $\lambda$

$\frac{h}{E_F}$	P		
	$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.15$
0.0	0.00	0.00	0.00
0.10	0.05	0.04	0.025
0.15	0.08	0.06	0.043
0.20	0.12	0.09	0.054
0.25	0.14	0.12	0.065
0.30	0.16	0.14	0.076
0.35	0.19	0.18	0.084
0.40	0.24	0.22	0.097
0.45	0.29	0.24	0.122
0.50	0.38	0.32	0.147
0.55	0.45	0.37	0.159
0.60	0.52	0.43	0.166
0.65	0.58	0.46	0.173

**Table T5**

**An evaluated result of critical temperature for balanced super fluid at unitarity  $T_{CO}$  and critical temperature  $T_{Cg}$  for balanced super fluid,  $T_{CO}$  is calculated from mean field theory and  $T_{Cg}$  is calculated using formalism Noziers-Schdmit-Rink correction<sup>16</sup>**

$\frac{\lambda}{v_F}$	$T_{CO}$	$T_{Cg}$
0.00	0.52	0.152
0.10	0.55	0.167
0.20	0.57	0.176
0.30	0.59	0.189
0.40	0.60	0.209
0.50	0.62	0.248
0.60	0.58	0.327
0.70	0.55	0.365
0.75	0.54	0.399
0.80	0.52	0.417

## CONCLUSION

In this paper, we have theoretically studied spin-orbit coupled fermionic condensate. We have evaluated various parameters as a function of SOC parameter. The evaluated parameters are tricritical temoeratre, tricritical magnetic field, tricritical polarization and two types of critical temperature  $T_{CO}$  and  $T_{Cg}$  all as a function of SOC parameter, Our theoretically obtained results show that all parameter in crease and decrease with SOC parameter.

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