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An Evaluation of Energy Per Particle of Bec (Bose-Einstein Condensate) as a Function of (T/T_c^o) Using Non-Interacting and Interacting Models

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ABSTRACT

Using non-interacting and interacting models, energy per particle of Bose Einstein Condensate (BEC) trapped in a harmonic potential were calculated. Our theoretical evaluation indicates that energy per particle in Bose-Einstein Condensate depends linearly with the interaction parameter η.

Keywords: non-interacting, interacting models, BEC, Harmonic potential, Energy pre particle, interaction parameter.

INTRODUCTION

In an earlier paper[1], we have presented a method of calculation of condensate fraction (N_o/N) as a function of (T/T_c^o) for interacting and non-interacting models with two different traps. In this paper, we have evaluated the energy per particle of BEC as a function of (T/T_c^o) using perturbation expansion[2] and self-consistent calculation based on Popov approximation[3]

MATHEMATICAL FORMULAS USED IN THE EVALUATION

Non-interacting Bose gas[4]

Considering trapped bosons at finite temperature the total energy of the system can be expressed as

$$E = \int_0^\infty d\varepsilon \rho(\varepsilon) \, \varepsilon / (e^{\beta\varepsilon} - 1) \quad (1)$$

Where $\rho(\varepsilon)$ is the density of state given by

$$\rho(\varepsilon) = \frac{1}{2}(\hbar\omega_{h0})^{-3}\varepsilon^2 \quad (2)$$

ω_{h0} =geometrical average of the oscillator frequencies

$$\omega_{h0} = (\omega_x\omega_y\omega_z)^{\frac{1}{3}} \quad (3a)$$

The transition temperature at which the condensation occurs

$$K_\beta T_c^{\,0} = 0.94\hbar\omega_{h0}N^{\frac{1}{3}} \quad (3b)$$

Where N is the total number of particles Using (2) in (1) the total energy of the system is given by

$$\frac{E}{NK_\beta T_c^0} = 3\xi(4) \Big/ \xi(3) \left(\frac{T}{T^0{}_c}\right)^4 \quad (4)$$

The Specific heat $C = \frac{\partial E}{\partial T}$,

Condensate fraction

$$N/N_\circ = \left(1 - T/T_c^\circ\right)^3 \quad (4a)$$

The specific heat of condensed Bose gas is given by

$$\frac{C}{NK_\beta} = \frac{12\xi(4)}{\xi(3)} \left(\frac{T}{T_c^0}\right)^3 \quad (5)$$

The entropy S is given by

$$\frac{S}{NK_\beta} = \frac{3\xi(4)}{\xi(3)} \left(\frac{T}{T_c^0}\right)^3 \quad (6)$$

On comparing the results with well-known theory of uniform Bose gases, we have

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T^0{}_c}\right)^{\frac{3}{2}} \quad (7a)$$

$$K_\beta T^0{}_c = (2\pi\hbar^2/m)[n/\xi(\tfrac{3}{2})]^{\frac{2}{3}} \text{ with } n=N/V \quad (7b)$$

$$\frac{E}{(NK_\beta T_c^0)} = 3\xi(\tfrac{5}{2}) \Big/ [2\xi(\tfrac{3}{2})]\left(\frac{T}{T^0{}_c}\right)^{\frac{5}{2}} \quad (7c)$$

$$\frac{C}{NK_\beta} = \frac{15}{2}\xi(\tfrac{5}{2}) / [2\xi(\tfrac{3}{2})]\left(\frac{T}{T^0{}_c}\right)^{\frac{3}{2}} \quad (7d)$$

$$\frac{S}{NK_\beta} = 3\xi(\tfrac{5}{2}) / [2\xi(\tfrac{3}{2})]\left(\frac{T}{T^0{}_c}\right)^{\frac{3}{2}} \quad (7e)$$

EFFECTS OF INTERACTION

By taking non-interacting gas model we have seen that the fraction of atoms in the condensate

$$\frac{N_o}{N} = 1 - \left(\frac{T}{T_c^\circ}\right)^3$$

Energy

$$E \propto T^4$$

Now, the question arises whether the prediction of ideal gas is adequate and under what condition the effect of interaction becomes sizable. The effects of two-body interactions 'in a dilute Bose gas are expected to be significant only in the presence of the condensate, since only in this case the density become relatively high due to the occurrence of the peak in the centre of the trap. A first important consequence of repulsive forces is the broadening of the condensate peak. This effect, at zero temperature, provides a dramatic change in the density distribution also at finite T and its experimental observation is an important evidence of the role played by two-body forces. The opposite happens in the presence of attractive forces, which produce a further narrowing of the peak and a consequent increase of the peak density.[5-12]

Let us discuss the effects of a repulsive interaction by estimating the relevant energies of the system. At zero temperature the interacting energy per particle can be simply estimated using the Thomas-Fermi approximation $\frac{E_{\text{int}}}{N} = (\frac{2}{7})\mu$ where $\mu = (\frac{1}{2})\hbar\omega_{h0}(15Na/a_{h0})^{\frac{2}{5}}$ is the value of the chemical potential. It is useful to compare $\frac{E_{\text{int}}}{N}$, or equivalently μ, with the thermal energy $K_\beta T$. If $K_\beta T$ is smaller than μ, then one expects to observe important effects in the thermodynamic behavior due to interactions. If instead $K_\beta T$ is larger than μ interactions will provide only by perturbative corrections. Thus, for repulsive force the chemical potential provides an important scale of energy lying between the oscillator energy and the critical temperature: $\hbar\omega_{h0} \ll \mu \ll K_\beta T_c^0$ A useful parameter is the ratio

$$\eta = \frac{\mu}{K_\beta T_c^0} = \alpha \left(N^{\frac{1}{6}} \frac{a}{a_{h0}}\right)^{\frac{2}{5}} \quad (8)$$

Between the Chemical potential calculated at T = 0 in Thomas-Fermi approximation and the critical temperature for non-interacting particles in the same trap. Here $\alpha = 15^{2/5}[\varsigma(3)]^{1/3}/2 \simeq 1.57$ is a numerical coefficient. If one uses the typical values for the parameters of current experiments, one finds that η ranges from 0.35 to 0.40. Thus one expects that interaction effects will also be visible at values of T of the order T_c°

In the absence of the condensate ($T>T_C$) interaction effects are less important because the system is very dilute. In this case, one can estimate the interaction energy using the expression $\frac{E_{\text{int}}}{N} = \frac{gN}{R_T^3}$ where $R_T = (2K_B T/m\omega_{ho}^2)^{1/2}$ is the classical radius of the thermal cloud. For temperatures of the order of T_c one finds

$$\frac{E_{int}}{NK_B T_c^\circ} \sim N^{1/6}\frac{a}{a_{ho}} \sim \eta^{5/2} \quad (9)$$

This ratio depends on the interaction parameter η through a higher power law as compared to the analogous ratio for the energy of the condensate, which is linear in η and the effect of E_{int} is hence much smaller for non condensed atoms.

Now, using the semi-classical picture, one can write

$$N_T = \int \frac{d\bar{r}\,d\bar{p}}{(2\pi\hbar)^3}\left\{Exp\left[\left({p^2}/{2m} + V_{eff}(\bar{r}) - \mu\right)/K_B T\right] - 1\right\}^{-1} \quad (10)$$

Using the Thomas-Fermi approximation for the effective mean field potential

$$V_{eff}(\bar{r}) - \mu = \left|V_{eff}(\bar{r}) - \mu\right| \quad (11)$$

This leads to the result

$$\frac{E}{NK_\beta T_c^\circ} = \frac{3\xi(4)}{\xi(3)}t^4 + \frac{1}{7}\eta(1-t^3)^{2/5}(5+16t^3) \quad (12)$$

Where $t = T/T_c^\circ$

DISCUSSION OF RESULTS

In this paper, we have evaluated the energy per particle of BEC trapped gas as a function of (T/T_c°) using non-interacting and interacting model. Using trapped non-interacting bosons at finite temperature and taking grand canonical ensemble the expression for condensate function (N_O/N) and total energy of the system have been evaluated. These are given in equation (4a) and (4) respectively. The interaction effect has been studied by introducing a parameter η which is the ratio of $(\mu/k_B T_c^\circ)$, μ is the chemical potential calculated at T = 0 in Thomas-Fermi approximation. In terms of η the expression for energy has been given in equation (l2). From our calculation it appears that energy E depend upon the interaction parameter η.

Table T_1

Evaluated results of Energy per particle, specific heat and entropy of trapped Bose Gas (non-interacting models).

T/T_c°	$E/NK_\beta T_c^\circ$	C/NK_β	S/NK_β
0.1	0.00027	0.01080	0.00270
6.2	0.00432	0.08642	0.02160
0.3	0.2187	0.29165	0.07291
0.4	0.06913	0.69133	0.17283
0.5	0.16878	1.35025	0.33756
0.6	0.34998	2.33327	0.58331
0.7	0.64839	3.70510	0.92227
0.8	1.10612	5.53062S	1.38266
0.9	1.77179	7.87466	1.96866

Table T_2

Comparison of results of $\left(E/_{NK_\beta T_c^\circ},\right)$, $\left(C/_{NK_\beta}\right)$ and $\left(S/_{NK_\beta}\right)$ for trapped Bose Gas and ideal Bose gas as function of T/T_c°

$T/_{T_c^\circ}$	Energy		Sp. heat		Entropy	
	$(\frac{E}{NK_\beta T_c^0})_{trapp}$	$\left(E/_{NK_\beta T_c^\circ}\right)_{idel}$	$(\frac{C}{NK_\beta})_{trapp}$	$\left(C/_{NK_\beta}\right)_{idel}$	$(\frac{S}{NK_\beta})_{trapp}$	$\left(S/_{NK_\beta}\right)_{idel}$
0.1	0.00027	0.002437	0.01080	0.06089	0.00270	0.02437
0.2	0.00432	0.01379	0.08642	0.172199	0.02160	0.06893
0.3	0.2187	0.03799	0.19165	0.316301	0.07291	0.12664
0.4	0.06913	0.077988	0.069133	0.487054	0.17283	0.19478
0.5	0.16878	0.136240	1.35025	0.68069	0.33756	0.27298
0.6	0.34998	0.214910	2.33323	0.89477	0.58331	0.35818
0.7	0.64839	0.31595	3.70510	1.12754	0.92227	0.45736
0.8	1.10612	0.441168	5.53062	1.37760	1.38266	0.55146
0.9	1.77179	0.59222	7.87466	1.64381	1.96866	0.65803
1.0	2.70049	0.77069	10.80199	1.92524	2.70049	0.77069

Table T_3

Evaluated result of energy per particle as function of $\left({}^{T}/_{T_c^\circ}\right)$ for η = 0.4 using equation (12) (Interacting Model)

${}^{T}/_{T_c^\circ}$	${}^{E}/_{Nk_\beta T_c^\circ}$
0.1	0.328
6.2	0.345
0.3	0.397
0.4	0.426
0.5	0.478
0.6	0.532
0.7	0.657
0.8	0.787
0.9	0.985

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