J. Pure Appl. & Ind. Phys. Vol.1 (2), 121-126 (2011)

Impact Ionization Effects on Propagation of a Millimeter Wave in GaAs

RAJENDRA PATHAK[1], J. PARASHAR[2] and S. KATAREY[3]

[1]Department of Mathematics, Samrat Ashok Technological Institute, Vidisha (M.P.) 464001, India
[2] Department of Physics, Samrat Ashok Technological Institute, Vidisha (M.P.) 464001, India
E mail: j.p.parashar@gmail.com
[3]Department of Mechanical Engineering, Samrat Ashok Technological Institute, Vidisha (M.P.) 464001, India.

ABSTRACT

Effect of impact ionization on propagation of a millimeter (mm) wave with a Gaussian profile is studied under paraxial ray approximation. It is found that early in time the charge density is less and hence the defocusing of mm wave is less however later in time and space as the carrier density builds up due to impact ionization the defocusing of the mm wave is rapid and this results in decrease in the intensity of the wave.

PACS Nos.: 52.40.Db; 52.25.Jm; 52.35.Hr; 41.20.Jb; 42.70.Na

Key Works: Impact ionization, semiconductors, microwaves

Interaction of electromagnetic wave with semiconductor is an active area of research for almost last four decades and several nonlinear effects viz. harmonic generation, instabilities, wave mixing, etc. have been observed[1,2]. When an electromagnetic wave passes through a semiconductor the valence electron can jump to conduction band at the expense of the wave energy via one of the following processes: impact ionization, tunnel ionization or avalanche effect[3]. Some notable applications of this transition are in switches, millimeter wave generation, UV lasers etc.[4,5]. In impact ionization an electron or hole can gain energy in the presence of electric field and subsequently lose their energy by creation of other charge carriers. This can lead to avalanche breakdown in semiconductors. The electron hole (e-h) plasma created via impact ionization is space time varying and alters the propagation dynamics of the electromagnetic wave.

In such processes the frequency range of incident radiation is important as transit time effect come into play. Amongst several semiconductor materials, Gallium Arsenide (GaAs), a direct band gap semiconductor, has some important applications such as in microwave frequency integrated circuits, infrared light emitting diodes, laser diodes, mobile phones, satellite communications, microwave point to point links, radar systems and solar cells[6,7]. In the microwave frequency range GaAs is one of the best candidates because of its higher saturated electron velocity, higher electron mobility, less noise as compared to Silicon (Si) and also can be operated at higher powers due to higher breakdown voltages[8,9]. In this communication we develop a theory to study the effect of impact ionization on propagation of a Gaussian millimeter wave propagating through GaAs. The wave equation is solved by generalizing the paraxial ray theory of nonlinear wave propagation.

Consider the propagation of a millimeter wave with Gaussian intensity profile through a GaAs semiconductor with <100> orientation,

$$\vec{E} = \vec{E}_0 (z, r, t)\, e^{-i(\omega t - kz)} \tag{1}$$

At z = 0

$$E_0 = A_0\, e^{-r^2/r_0^2}, \quad for\ 0 < t < \tau \qquad = 0 \quad , \quad otherwise \tag{2}$$

where τ is the pulse duration.

The wave creates electron – hole (e – h) pairs inside semiconductor via impact ionization. The ionization rates for electrons and holes can be written as

$$\frac{\partial n_e^2}{\partial t} = \alpha_i\, n_{e0}^2 \ , \tag{3}$$

and

$$\frac{\partial n_h^2}{\partial t} = \beta_i\, n_{h0}^2 \ , \tag{4}$$

respectively. Here $n_{e(h)}$ is electron (hole) density, $n_{e(h)0}$ being the equilibrium density, and $\alpha_i\ (\beta_i)$ is electron (hole) ionization rate given by

$$\alpha_i = \alpha_0\, e^{-E_{0n}/E}, \tag{5}$$

$$\beta_i = \beta_0\, e^{-E_{0p}/E}, \tag{6}$$

$\alpha_0, \beta_0, E_{0n}$ *and* E_{0p} are constants (c.f. Shur[9] pp.188).

The electron (hole) current density $\vec{J}_{1e(h)}$ is governed by

$$\frac{d\vec{J}_{1e(h)}}{dt} + \nu\, \vec{J}_{1e(h)} = \frac{n_{e(h)} e^2 \vec{E}}{m^*_{e(h)}}, \tag{7}$$

where ν is the electron – hole collision frequency and $m^*_{e(h)}$ is the electron (hole) effective mass.

The millimeter wave field in the space – time evolving electron – hole plasma could be written as

$$\vec{E} = \vec{A}\, e^{i\phi}, \tag{8}$$

where $\vec{A}$ is a slowly varying function of z,t and ϕ is a fast varying function of *z, t*.

The wave equation governing the propagation of millimeter wave is written as[10, 11],

$$\nabla^2 \vec{E} - \frac{\varepsilon_L}{c^2} \frac{\partial \vec{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial \vec{J}}{\partial t}, \tag{9}$$

where ε_L is lattice permittivity and we have neglected the $\nabla(\nabla.\vec{E})$ term for transverse waves. For $\vec{E} = \hat{y} E_y$ form Eq.(8) we have

$$\frac{\partial^2 E_y}{\partial z^2} \approx i\frac{\partial^2 \phi}{\partial z^2} A e^{i\phi} - \left(\frac{\partial \phi}{\partial z}\right)^2 A e^{i\phi} + 2i\frac{\partial \phi}{\partial z}\frac{\partial A}{\partial z} e^{i\phi}, \quad (10)$$

$$\frac{\partial^2 E_y}{\partial t^2} \approx i\frac{\partial^2 \phi}{\partial t^2} A e^{i\phi} - \left(\frac{\partial \phi}{\partial t}\right)^2 A e^{i\phi} + 2i\frac{\partial \phi}{\partial t}\frac{\partial A}{\partial t} e^{i\phi}, \quad (11)$$

$$\nabla_\perp^2 E_y = \nabla_\perp^2 A e^{i\phi}, \quad (12)$$

where we have neglected $\partial^2 A/\partial z^2$ terms. Defining ω, k as $\omega = -\partial\phi/\partial z$ and $k = \partial\phi/\partial t$, with $\omega^2 = \omega_p^2(r=0) + k^2c^2$, and using Eqs.(7), (11) & (12) in Eq.(9) we get

$$\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{2i\omega\varepsilon_L}{c^2}\frac{\partial A}{\partial t} + \left(i\frac{\partial k}{\partial z} + i\frac{\varepsilon_L}{c^2}\frac{\partial \omega}{\partial t}\right)A - \frac{1}{c^2}\left[\omega_{pe}^2 + \omega_{ph}^2 - \left(\omega_{pe}^2 + \omega_{ph}^2\right)\Big|_{r=0}\right]A = 0, \quad (13)$$

where $\omega_{pe(h)}^2 = 4\pi n_{e(h)}/m_{e(h)}^*$ is the electron (hole) plasma frequency and $m_{e(h)}^*$ is the electron (hole) effective mass.

Multiplying Eq.(13) by A, we obtain

$$\frac{\partial A^2}{\partial t} + v_g\frac{\partial A^2}{\partial z} - \frac{ic^2}{\omega}A\nabla_\perp^2 A + \frac{A^2}{2\omega^2}\frac{\partial(\omega_{pe}^2 + \omega_{ph}^2)}{\partial t} + \frac{i}{\omega}\left[\omega_{pe}^2 + \omega_{ph}^2 - \left(\omega_{pe}^2 + \omega_{ph}^2\right)\Big|_{r=0}\right]A^2 = 0. \quad (14)$$

Later we will recast Eq.(14) in terms of new variables z', r, t' where $t' = t - z/v_g$, $z' = z$.

We assume a Gaussian ansatz for the r-profile of laser intensity[12],

$$|E_0|^2 = \frac{E_{00}^2}{f^2} e^{-r^2/r_0^2 f^2}, \quad (15)$$

with f as beam width parameter. Using Eq.(15) we expand Eqs.(3) and (4) around r=0 and obtain

$$\frac{\partial \omega_{pe}^2}{\partial t} = \alpha_0\,\omega_{pe0}^2\left[Q_e(r=0) + \frac{\partial Q_e}{\partial r^2}\Big|_{r=0} r^2\right], \quad (16)$$

and

$$\frac{\partial \omega_{ph}^2}{\partial t} = \beta_0\,\omega_{ph0}^2\left[Q_h(r=0) + \frac{\partial Q_h}{\partial r^2}\Big|_{r=0} r^2\right], \quad (17)$$

respectively.

Here

$$Q_{e(h)} = \exp[-(E_{0n(p)}/E_{00})(f\,e^{r^2/2r_0^2 f^2})].$$

On integrating Eqn.(16) and (17) we get

$$\omega_{pe}^2 = \int_{z/v_g}^{t+z/v_g}\left(\alpha_0' - \alpha_0'' r^2/r_0^2\right)dt', \quad (18)$$

and

$$\omega_{ph}^2 = \int_{z/v_g}^{t+z/v_g}\left(\beta_0' - \beta_0'' r^2/r_0^2\right)dt', \quad (19)$$

respectively.

Here,

$$\alpha_0'' = \frac{\alpha_0'}{2f}\frac{E_{0n}}{E_{00}}, \quad \alpha_0' = \alpha_0\,\omega_{pe0}^2\, e^{-(E_{0n}/E_{00})f},$$

$$\beta_0'' = \frac{\beta_0'}{2f}\frac{E_{0n}}{E_{00}},$$

$$\beta_0' = \beta_0\,\omega_{ph0}^2\, e^{-(E_{0n}/E_{00})f}.$$

Using Eqs.(18) & (19) in Eq.(14) we get

$$2ik\frac{\partial A}{\partial z'} + \nabla_\perp^2 A + \frac{(\alpha^p + \beta^p)}{c^2}\frac{r^2}{r_0^2}A = 0. \quad (20)$$

Introducing an eikonal

$A = A_0(r,z)\exp[-S(r,z)]$ [13]and separating the real and imaginary parts of above equation we obtain

$$2\frac{\partial S}{\partial z'}+\frac{1}{k}\left(\frac{\partial S}{\partial r}\right)^2=\frac{1}{k\,A_0}\nabla_\perp^2 A_0+\frac{(\alpha^p+\beta^p)r^2}{r_0^2\,k\,c^2} \tag{21}$$

and

$$\frac{\partial A_0^2}{\partial z'}+\frac{1}{k}\left(\nabla_\perp^2 S\right)A_0^2+\frac{1}{k}\frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r}=0 \tag{22}$$

respectively.

Following Eq.(15) we write

$$A_0^2=\frac{A_{00}^2}{f^2}\exp(-r^2/r_0^2 f^2), \tag{23}$$

and expand *S* as

$$S=\frac{k}{2}\beta(z')r^2+\phi(z'), \tag{24}$$

in the paraxial ray approximation. Using Eqs.(23) &(24) in Eqs.(21) & (22) we get

$$\beta=\frac{1}{f}\frac{df}{dz'}, \tag{25}$$

and the equation governing the beam width parameter *f* as

$$\frac{d^2 f}{dz'^2}=\frac{1}{R_d^2 f^3}+\frac{(\alpha^p+\beta^p)f\,r_0^2}{R_d^2 c^2}, \tag{26}$$

where $R_d^2=k\,r_0^2$.

We introduce dimensionless variables $\xi=z'/R_d$ and $\eta=10^{-2}\frac{\omega_{pe0}^2 r_0^2 t'}{2c^2}$, then Eq.(26) could be rewritten as

$$\frac{d^2 f}{d\xi^2}=\frac{1}{f^3}+f\,.10^2.$$

$$\int_0^\eta \frac{1}{f}\left[\frac{\alpha_0 E_{0n}}{E_{00}}e^{-\frac{E_{0n}}{E_{00}}f}+\frac{\beta_0 E_{0p}}{E_{00}}\frac{\omega_{ph0}^2}{\omega_{pe0}^2}e^{-\frac{E_{0p}}{E_{00}}f}\right]d\eta \tag{27}$$

We have solved Eq.(27) numerically for $E_{0n}/E_{00}=6.27$, $E_{0p}/E_{00}=4.82$, $n_e\approx n_h\approx 5\times10^6\,cm^{-3}, \varepsilon_L\sim 11$, $\lambda=1cm, r_0=1mm$, $m_e^*=0.063m_0, m_h^*=0.063m_0$, $m_0=9.1\times10^{-31}kg$, $\alpha_0=2.19\times10^6$, $\beta_0=2.47\times10^6$ with the boundary conditions: f =1 and $df/d\xi=0$ at ξ=0 for all η. We have chosen time step size $\Delta\eta=0.01$ and the space step size $\Delta\xi=0.01$. We write Eq.(27) as

$$f''=\frac{1}{f^3}+P\,f \tag{28}$$

where

$P=10^2$.

$$\int_0^\eta \frac{1}{f}\left[\frac{\alpha_0 E_{0n}}{E_{00}}e^{-\frac{E_{0n}}{E_{00}}f}+\frac{\beta_0 E_{0p}}{E_{00}}\frac{\omega_{ph0}^2}{\omega_{pe0}^2}e^{-\frac{E_{0p}}{E_{00}}f}\right]d\eta$$

and the prime represents differentiation with respect to ξ. We begin by evaluating P at ξ=0 for all values of η. We solve Eq.(28) with the Runge-Kutta method using the value of P obtained at ξ=0. Using these values of *f*, we evaluate P at $\xi=\Delta\xi$ at different values of η. This way we advance in ξ. In Fig. 1 we have plotted the beam width parameter *f* and in Fig.2 the axial intensity $I=I_0/f^2$ as a function of ξ for different values of η. Early in time, the axial intensity decreases due to diffraction divergence. However, as the e-h density builds up the effect of self-defocusing becomes important and intensity falls off more rapidly with the distance of propagation. It can be seen that the millimeter wave can be severely defocused due to impact ionization.

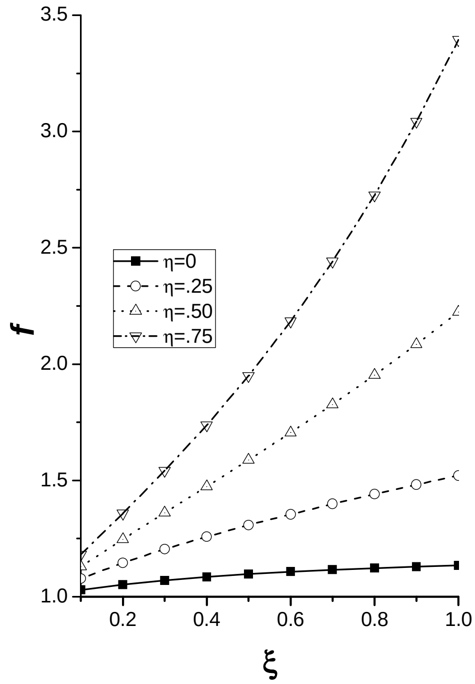

Fig. 1. Variation of beam width parameter *f* with ξ.

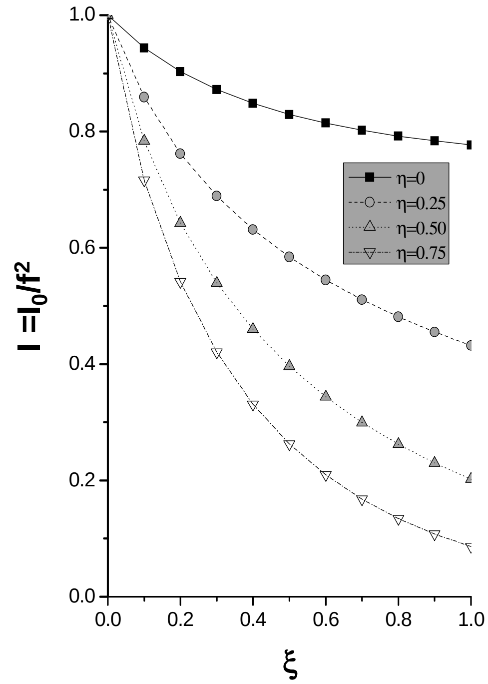

Fig.2. Variation of axial intensity $I = I_0 / f^2$ **with ξ.**

REFERENCES

1. Shen Y. R.: *The Principles of Nonlinear Optics* (Wiley, New York 1984).
2. Singh L., Konar S. and Sharma A. K., *J. Phys. D : Appl. Phys*. 34, 2237 (2001).
3. Vavilov V. S., *Sov. Phys. Usp*. 37, 269 (1994).
4. Basov N. G., Grasyuk A.Z., Zubarev I.G., Katulin V. A., and Krokhin O. N., *Sov. Phys. JETP* 23, 366 (1966).
5. Liu C. S. and Tripathi V. K., *J.Appl. Phys*. 83, 15 (1998).
6. Bhattacharya P: *Semiconductor Optoele-ctronic Devices* (Pearson Education, New Delhi, 2001).
7. Tyagi M. S.: *Introduction to Semicond-uctor Materials and Devices* (Wiley, New York, 2000).
8. Sze S. M.: *Physics of Semiconductor Devices* (Wiley, New York, 2001).
9. Shur M.: *Physics of Semiconuctor Devices* (Prentice Hall India, New Delhi, 2004).
10. Jackson J. D.: *Classical Electrodyn-amics* 3ed (John Wiley, New York, 2004).
11. Parashar J., Pandey H. D., and Tripathi V.K., *Phys. Plasmas* 4, 3040 (1997).

12. Akhmanov S. A., Sukhorukov A. P., and Khokholov R.V., *Sov. Phys. Usp*. 10, 669 (1968).
13. Liu C.S., and Triptahi V. K.: *Interaction of Electromagnetic Waves with Electron Beams and Plasmas* (World Scientific, Singapore, 1994).