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A Theoretical Study of Magnetic Properties of Parabolic Quantum Dots in the Presence of Spin-orbit Interaction

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ABSTRACT

Using the theoretical formalism of **O. Voskoboynikov *et al*.[J Appl. Phys., 94, 5891 (2003)],[J. Appl. Phys. 59, 1 (2000)] and [Phys. Rev. B63, 165306 (2001)],** we have theoretically evaluated magnetization and magnetic susceptibility of parabolic quantum dots in the presence of spin orbit interaction. We observe the following facts:

Our theoretical analysis indicate that magnetization and magnetic susceptibility show quite interesting properties at low temperature. One observes abrupt change of the magnetization and susceptibility at low magnetic fields.

This type of physical behavior is due to the alternative level crossing between spin-split electron levels in the energy spectrum. This is essentially due to spin-orbit interaction.

Our calculation also demonstrate that if one uses the parameter of InAs semiconductor quantum dots then one observes the enhancement of the par magnetism of the dots. This effect can be controlled by the effect of external electric fields or the dot design.

The theoretical analysis of the paper also reveals the fact that magnetic properties are elegantly can be studied with the help of parabolic QDs. It is the parabolic (cylindrical coordinate ((ρ,ϕ)) which demonstrates the dynamics of the charge carriers in the dots. In this way, the work reported in this paper will be quite helpful in the area of instrument design using parabolic QDs.

Keywords: Parabolic quantum dots, Spin-orbit interaction, Spin degeneracy of the energy levels, Spin splitting, Enhancement of par magnetism, Paschen- Back rffect, Rashba interaction term.

INTRODUCTION

Nanostructure materials are classified into three main categories namely Quantum Well (QW), Quantum wire (QWR) and Quantum Dots (QD). Quantum well is 2D electronic system. Here the motion of the charge carriers is restricted in one dimension but they are free to move in two dimension. In QWR, the motion of the charge carriers is restricted in two dimension and they are free to move in one dimension. In QD, The motion of the charge carriers is restricted in all the three dimensions. They are localized to a particular point. QD has wide applications. It has the ability to manipulate and control processes that involve transition between different spin states. Now days it is widely used in quantum computation and quantum communications. Parabolic QDs are made of narrow band gap materials of InAs and GaAs. Parabolic QDs are suitable to study spin relaxation of conduction electrons. It is also helpful in the study of phonon modulation of the spin-orbit interaction as relaxation process[1-8].

The electron spin controls design of the energy shells and magnetic properties of semiconductor QDs[9]. Among the other spin dependent interactions, the spin orbit interaction (the interaction between orbital angular and spin momenta) plays an observable role in the energy spectrum formation for III-V semiconductor nanostructures. When the potential through which the carriers move is inversion symmetric one, the sin-orbit interaction removes the spin degeneracy of the energy levels without external magnetic fields. It sufficiently alters the electronic properties of semiconductor nano-structures[10-12].

In this paper, using the theoretical formalism of O Voskoboynikov *et al*.[13-15], we have theoretically evaluated the magnetization and magnetic susceptibility of InAs parabolic quantum dots with and without spin-orbit (SO) interaction. The calculations are performed for parabolic QD with N=1, 2, 3, 4, 5, 6 electron systems taken into account with and without SO. In this model calculation, one has focused the Rashba term in the spin-orbit interaction potential. We have not included the other spin-interaction term Dresselhaus in the calculation. We observed that due to spin-orbit interaction term there is abrupt change in the magnetization and magnetic susceptibility at low magnetic fields. This type of change is considered due to the spin splitting at zero magnetic field. This leads to crossing of the energy levels in weak external electric field or the dot design. We have evaluated temperature dependent magnetization for N=4 system with four different temperatures with and without SO. We have also calculated temperature dependent magnetic susceptibility for N=4 electron systems with and without SO. Our theoretically evaluated results are in good agreement with other theoretical workers[16,17].

MATHEMATICAL FORMULAE USED IN THE STUDY

Model of the Quantum dot

The single particle Hamiltonian in the lateral cylindrical coordinate $(\rho\phi)$ in the presence of uniform magnetic field B applied along the axis of the dot (z direction) is written as

$$H = \frac{-\hbar^2}{2m(E)}[\frac{\partial}{\rho\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}]-\frac{i}{2}\hbar\omega_c(E,B)\frac{\partial}{\partial\phi}+\frac{1}{2}m(E)\omega^2{}_c(E,B)$$
$$\rho^2+V_c(\rho)+V^R{}_{so}(\rho,\phi)+\frac{1}{2}\sigma_z\mu_B g(E)B \tag{1}$$

$$V_c(\rho)=\frac{1}{2}m(E)\omega^2{}_0\rho^2 \tag{2}$$

Here, $V_c(\rho)$ is the effective lateral confinement potential, $\hbar\omega_0$ is the characteristic confinement energy. The electron effective mass is given by[18,19]

$$\frac{1}{m(E)}=\frac{1}{m(0)}\frac{E_g(E_g+\Delta)}{(3E_g+2\Delta)}[\frac{1}{E+E_g}+\frac{1}{E+E_g+\Delta}] \tag{3}$$

Here, E denotes the electron energy in the conduction band, m(0) is the conduction-band – edge effective mass, E_g and Δ are the main band gao and the spin-orbit band splitting respectively. $\omega_c(E,B)$ is the electronic cyclotron frequency and σ_z is the Pauli spin z matrix. The electronic cyclotron frequency is gien by

$$\omega_c(E,B)=\frac{eB}{m(E)} \tag{4}$$

The effective Lande factor of the semiconductor is given by

$$\mathrm{g(E)}=2[1-\frac{m_0}{m(E)}\frac{\Delta}{3(E+E_g)+2\Delta}] \tag{5}$$

Here, e is the electron charge and m_0 is the frtee electron mass and $\mu_B=\frac{e\hbar}{2m_0}$ is the Bohr magneton. The Rashba spin-orbit interaction term in equation (1) is given by[20,21]

$$V^R{}_{so}(\rho,\phi)=\sigma_z\alpha\frac{dV_c(\rho)}{d\rho}(k_\phi{}^{\rightarrow}+\frac{e}{2\hbar}B_\rho) \tag{6}$$

Where $k_\phi{}^{\rightarrow}=-i(1/\rho)\frac{\partial}{\partial\phi}$ and α is the spin-orbit coupling parameter within the Rashba approach. The eigen energies of the Hamiltonian can be obtained by means of a self-consistent solution of the following equation[22]

$$E_{n,l,\sigma} = \hbar\Omega_\sigma(E_{n,l,\sigma}, B)(2n+|l|+1) + l\frac{\hbar\omega_c(E_{n,l,\sigma}, B)}{2} + s[\frac{\mu_B}{2}g(E_{n,l,\sigma})B + l\alpha m(E_{n,l,\sigma})\omega^2{}_0] \quad (7)$$

Where

$$\Omega^2{}_\sigma(E,B) = \omega^2{}_0 + \frac{\omega^2{}_c(\mathrm{E},B)}{4} + s\alpha\frac{m(E)\omega^2{}_0}{\hbar}\omega_c(E,B) \quad (8)$$

Here, n,l and $s=\pm1$ refer to the main quantum number, orbital quantum number and the electron spin polarization along the z axis correspondingly. The electron energy levels in equation (7) with different spins and the same angular momentum $|l|>0$ due to the spin-orbit interaction are split at B=0 and cross with increasing of the magnetic field. The levels with parallel spins and angular momentum (antiparallel spin and angular momentum) remain twofold degenerated. This is well known Kramers degeneracy. The first crossing point for the lowest spin-split levels $(|l|=1)$ is determined by

$$\frac{\Phi}{\Phi_0} \approx \frac{\Delta E}{\hbar\omega_0} [=1 \quad (9)$$

Where Φ is the magnetic flux in the dot area, Φ_0 is the magnetic flux quanta and ΔE is the energy spin splitting at B=0. The second crossing point occurs at

$$\frac{\Phi}{\Phi_0} = \frac{2\Delta E}{(2\Delta E + g\hbar\omega_0)} \quad (10)$$

Now, one generally fix only the thermal average of the total electron number to a given value N. In the case of fixed number of electrons, one uses the canonical ensemble description[23]. The thermal average of the total magnetization M and magnetic susceptibility χ of the system connected to reservoir and with fixed chemical potential are given by

$$M = \Sigma_{n,l,\sigma}(-\frac{\partial E_{n,l,\sigma}}{\partial B})f(E_{n,l,\sigma}-\xi) \quad (11)$$

$$\chi = \frac{\partial M}{\partial B} \quad (12)$$

Here, $f(E)$ is the Fermi distribution function and ξ is the chemical potential of the system determined by the following equation

$$N = \Sigma_{n,l,\sigma} f(E_{n,l,\sigma}-\xi) \quad (13)$$

DISCUSSION OF RESULTS

Using the theoretical formalism of O. Voskoboyinikov[13-15], we have theoretically studied the magnetic properties of parabolic quantum dot in the presence of spin-orbit interaction. We have evaluated theoretically the effect of the spin-orbit (SO) interaction on the electron magnetism and magnetic susceptibility of small semiconductor quantum dots. These characteristics demonstrate quite interesting behavior at low temperature. Spin- orbit interaction is considered as an analog of the general Paschen-back effect. One witnesses abrupt changes of the magnetization and susceptibility at low magnetic fields. In **table T1**, we have shown the evaluated result of the magnetization M / μ^{*}_{B} of InAs parabolic quantum dot with and without spin-orbit interaction for one and two electron system. The evaluation is performed as a function of $(\omega_C / \omega_0) x10^{-2}$. Here, ω_C is the electron cyclotron frequency, ω_0 is the confinement frequency, $\mu^{*}_{B} = \frac{e\hbar}{2m(0)}$ is the Bohr magneton, m(0) is the free electron mass. Our theoretically obtained results show that magnetization decrease as a function of (ω_C / ω_0) for both N=1 and N^{SO} =1. However, for N=2 and N^{SO}=2 trend is found differently. In this case, the values decrease and decrease negatively as a function of (ω_C / ω_0). In **table T2**, we repeated the calculation for N=3 and N^{SO}=3 and N=4 and N^{SO}=4 respectively as a function of (ω_C / ω_0). We observe in this that for N=3 and N^{SO}=3, the value of magnetization increase with (ω_C / ω_0). Similarly for N=4 the value decrease and N^{SO}=4 the value increase with (ω_C / ω_0). These results show that spin-orbit splitting partially lifts up the degeneracy and change the electron structure. There is a level crossing between the spin split electron levels in the energy spectrum. The level crossing provides a sharp jump in the magnetization. In **table T3**, we repeated the calculation of magnetization for N=5 and 6 electron systems with and without SO. In this case, for N=5 and N^{SO} =5, our evaluated results show that the magnetization decrease with $\frac{\omega_C}{\omega_0}$ for both the cases. In case of N=6 and N^{SO}=6, our obtained results indicate that the values decrease negatively for both the cases. These results also attribute the level of levels crossing. In **table T4,** we have shown the magnetization as a function of $\frac{\omega_C}{\omega_0}$ for N=4 electron system without SO. The results were taken for four different temperature namely T=0K, 0.5K, 1K and 4K respectively. In this case, we observe that the temperature dependent magnetization increase with $\frac{\omega_C}{\omega_0}$ for all the four temperature. Our obtained value is larger at T=0K and smaller for 4K. In **table T5**, we repeated the similar calculation for N^{SO}= 4 electron system for similar four temperatures. Here, we observe the similar trend as observed for N=4 with the value larger for 0K than 4K. The physical behavior of such type of result is the following: At low but finite temperature $K_\beta T \le \hbar\omega_0$, the magnetization follows the well known route. The totally occupied shells keep provide diamagnetic properties of the system and partially filled shells demonstrate paramagnetic properties. After that the peak value decrease exponentially $\exp[(-K_{\beta}T/\hbar\omega_0)]$ and magnetism approaches the Landau diamagnetism limit[24] when $K_{\beta}T \leq \hbar\omega_0$. When the magnetic field increases the magnetization demonstrate the paramagnetic peak. The spin-orbit interaction shifts the position of the peak. In **table T6,** we have presented the result of temperature dependent magnetic susceptibility χ (eVT^{-2}) as a function of $(\frac{\omega_C}{\omega_0})$ x10^{-2} for InAs parabolic quantum dots with four electron system without SO. for temperatures T=0.5K, 1K, 2K and 4K respectively. Our theoretically evaluated results indicate that χ (eVT^{-2}) decrease as a function of $(\frac{\omega_C}{\omega_0})$ x10^{-2} for each temperature. The magnitude is large for T=0.5K and small for T=4K. In **table T7,** we repeated the calculation of magnetic susceptibility for N=4 with SO. In this case, we observe that the value increases and decrease with $(\frac{\omega_C}{\omega_0})$ x10^{-2}. This type of behavior is also due to spin-orbit interaction. There is some recent[25-3o] results which also reveals the similar behavior.

Table T1: An evaluated result of magnetization M/ μ^*_B of Insb parabolic quantum dot as a function of $\frac{\omega_c}{\omega_0}$ for one and two electron systems with and without spin-orbit (SO) interaction, $\mu^*_B = \frac{e\hbar}{2m(0)}$ is the Bohr magneton, ω_c is the electron cyclotron frequency, m(0) is the free electron mass, ω_0 is the characteristic confinement frequency.

$\frac{\omega_c}{\omega_0}$ (10^{-2})	<------- M/ μ^*_B --------------------→			
	N=1	N^{SO} =1	N=2	N^{SO} =2
0	0.25	0.22	0.000	0.000
2	0.20	0.19	-0.004	-0.008
4	0.18	0.17	-0.006	-0.02
6	0.15	0.16	-0.07	-0.03
8	0.13	0.14	-0.10	-0.05
10	0.12	0.12	-0.12	-0.08
12	0.10	0.10	-0.14	-0.13
14	0.09	0.08	-0.17	-0.17
15	0.07	0.06	-0.19	-0.22
18	0.06	0.04	-0.21	-0.24
20	0.05	0.02	-0.23	-0.25
22	0.04	0.01	-0.24	-0.27
24	0.03	0.009	-0.26	-0.29
26	0.02	0.007	-0.28	-0.31
28	0.01	0.003	-0.30	-0.33
30	0.005	0.001	-0.32	-0.35

Table T2 : An evaluated result of magnetization M/ μ^*_B of InSb parabolic quantum dot as a function of $\frac{\omega_c}{\omega_0}$ for three and four electron systems with and without spin-orbit (SO) interaction, the terms used have usual meaning

$\frac{\omega_c}{\omega_0}$ (10^{-2})	<------- M/ μ^*_B -------------------->			
	N=3	N^{SO} =3	N=4	N^{SO} =4
0	1.20	0.23	2.02	0.23
2	1.00	0.45	1.89	0.50
4	0.90	0.62	1.78	0.55
6	0.80	0.70	1.70	0.60
8	0.70	0.84	1.62	0.65
10	0.60	0.97	1.58	0.72
12	0.50	1.03	1.52	0.76
14	.40	1.04	1,47	0.87
15	0.30	1.05	1.43	0.92
18	0.20	1.06	1.40	1.06
20	0.10	1.08	1.36	1.14
22	0.08	1.09	1.32	1.20
24	0.05	1.12	1.30	1.22
26	0.03	1.13	1.28	1.30
28	0.02	1.14	1.26	1.36
30	0.01	1.15	1.24	1.47

Table T3 : An evaluated result of magnetization M/ μ^*_B of InSb parabolic quantum dot as a function of $\frac{\omega_c}{\omega_0}$ for five and six electron systems with and without spin-orbit (SO) interaction, the terms used have usual meaning

$\frac{\omega_c}{\omega_0}$ (10^{-2})	<------- M/ μ^*_B -------------------->			
	N=5	N^{SO} =5	N=6	N^{SO} =6
0	1.42	1.30	0.000	0.000
2	1.37	1.25	-0.24	-0.20
4	1.32	1.20	-0.42	-0.36
6	1.30	1.17	-0.53	-0.48
8	1.26	1.14	-0.59	-0.55
10	1.24	1.14	-0.64	-0.63
12	1.20	1.12	-0.67	-0.69
14	1.16	1.10	-0.69	-0.72
15	1.14	0.92	-0.74	-0.79
18	1.12	0.86	-0.80	-0.83
20	1.00	0.80	-0.85	-0.87
22	0.92	0.75	-0.93	-0.95
24	0.84	0.70	-0.99	-1.05
26	0.76	0.67	-1.02	-1.09
28	0.52	0.54	-1.05	-1.12

30	0.43	0.48	-1.12	-1.16

Table T4 : An evaluated result of temperature dependent magnetization M/μ^*_B as a function of (ω_c/ω_0) x10^{-2} for a parabolic quantum dot for four electron system without spin-orbit interaction (SO)

(ω_c/ω_0) x10^{-2}	< -------- M/μ^*_B ------------------→(N=4)			
	T=0K	T=0.5K	T=1K	T=4K
0	0.000	0.000	0.000	0.000
5	0.222	0.204	0.196	0.154
7	0.245	0.223	0.207	0.183
9	0.297	0.276	0.255	0.209
12	0.334	0.327	0.307	0.274
14	0.375	0.358	0.342	0.305
16	0.423	0.409	0.395	0.326
18	0.489	0.475	0.455	0.406
20	0.556	0.522	0.508	0.483
22	0.592	0.556	0.542	0.507
24	0.645	0.593	0.577	0.524
26	0.708	0.725	0.708	0.678
28	0.757	0.739	0.726	0.706
30	0.805	0.822	0.817	0.809
32	0.871	0.924	0.908	0.895
34	0.978	0.997	0.975	0.946
36	1.524	1.487	1.325	1.207

Table T5: An evaluated result of temperature dependent magnetization M/μ^*_B as a function of (ω_c/ω_0) x10^{-2} for a parabolic quantum dot for four electron system with spin-orbit interaction (SO) (N^{SO}=4)

(ω_c/ω_0) x10^{-2}	< -------- M/μ^*_B ------------------→(N^{SO}=4)			
	T=0K	T=0.5K	T=1K	T=4K
0	0.000	0.000	0.000	0.000
5	0.222	0.204	0.196	0.154
7	0.245	0.223	0.207	0.183
9	0.297	0.276	0.255	0.209
12	0.334	0.327	0.307	0.274
14	0.375	0.358	0.342	0.305
16	0.423	0.409	0.395	0.326
18	0.489	0.475	0.455	0.406
20	0.416	0.522	0.508	0.483
22	0.455	0.556	0.542	0.507
24	0.520	0.593	0.577	0.524
26	0.547	0.725	0.708	0.678
28	0.596	0.739	0.726	0.706
30	0.625	0.822	0.817	0.809
32	0.647	0.924	0.908	0.895
34	0.708	0.997	0.975	0.946

36	0.759	1.487	1.325	1.207

Table T6 : An evaluated result of temperature dependent susceptibility $\chi(\mathrm{eVT}^{-2})$ as a function of $\frac{\omega_C}{\omega_0}x10^{-2}$ for InAs parabolic quantum dot with four electrons without SO

$\frac{\omega_C}{\omega_0}x10^{-2}$	<------ $\chi(eVT^{-2})$ ------------------------→			
	T=0.5K	T=1K	T=2K	T= 4K
0.00	0.0452	0.0276	0.0142	0.0105
0.20	0.0423	0.0243	0.0126	0.0094
0.40	0.0378	0.0208	0.0108	0.0086
0.60	0.0356	0.0187	0.0095	0.0076
0.80	0.0328	0.0165	0.0084	0.0067
1.00	0.0307	0.0143	0.0076	0.0056
1.50	0.0284	0.0127	0.0065	0.0048
2.00	0.0255	0.0108	0.0057	0.0041
2.25	0.0239	0.0095	0.0042	0.0036
2.50	0.0216	0.0087	0.0035	0.0032
2.60	0.0186	0.0064	0.0030	0.0031
2.70	0.0167	0.0052	0.0026	0.0026
2.80	0.0142	0.0045	0.0024	0.0024
3.00	0.0134	0.0036	0.0020	0.0018
3.50	0.0115	0.0025	0.0018	0.0014
4.00	0.0108	0.0015	0.0015	0.0010
4.50	0.0092	0.0008	0.0010	0.0008
5.00	0.0087	0.0006	0.0005	0.0006
5.50	0.0052	0.0004	0.0003	0.0004

Table T7: An evaluated result of temperature dependent susceptibility $\chi(\mathrm{eVT}^{-2})$ as a function of $\frac{\omega_C}{\omega_0}x10^{-2}$ for InAs parabolic quantum dot with four electrons with SO (N^{SO} =4)

$\frac{\omega_C}{\omega_0}x10^{-2}$	<------ $\chi(eVT^{-2})$ ------------------------→			
	T=0.5K	T=1K	T=2K	T= 4K
0.00	0.000	0.000	0.000	0.000
2.00	0.008	0.009	0.008	0.004
4.00	0.010	0.010	0.010	0.005
6.00	0.012	0.015	0.012	0.006
8.00	0.014	0.020	0.014	0.007
10.0	0.018	0.024	0.020	0.008
12.00	0.020	0.017	0.022	0.009
14.00	0.022	0.012	0.015	0.010
16.00	0.024	0.010	0.012	0.012
18.00	0.026	0.008	0.010	0.014
20.00	0.028	0.005	0.008	0.009
22.00	0.030	0.004	0.007	0.006
26.00	0.010	0.003	0.006	0.005
28.00	0.005	0.002	0.005	0.003

30.00	0.004	0.001	0.004	0.001

CONCLUSION

From the above theoretical investigations and analysis, we have come across the following conclusion;

(1) Our theoretical analysis indicate that magnetization and magnetic susceptibility show quite interesting properties at low temperature. One observes abrupt change of the magnetization and susceptibility at low magnetic fields.

(2) This type of physical behavior is due to the alternative level crossing between spin-split electron levels in the energy spectrum. This is essentially due to spin-orbit interaction.

(3) Our calculation also demonstrate that if one uses the parameter of InAs semiconductor quantum dots then one observes the enhancement of the par magnetism of the dots. This effect can be controlled by the effect of external electric fields or the dot design.

(4) The theoretical analysis of the paper also reveals the fact that magnetic properties are elegantly can be studied with the help of parabolic QDs. It is the parabolic (cylindrical coordinate ((ρ, ϕ)) which demonstrates the dynamics of the charge carriers in the dots. In this way, the work reported in this paper will be quite helpful in the area of instrument design using parabolic QDs.

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