

Convective Heat Transfer in a Non-uniformly Vertical Channel with Hall Effects

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ABSTRACT

We analyse the effect of Hall currents on the Convective Heat Transfer flow of a viscous electrically conducting fluid in a vertical channel bounded by the flat plates at $x = \pm L$. Which are maintained at Non uniform Temperature in the presence of Heat sources taking the slope δ of the boundary temperature the Non-linear coupled equations governing the flow and Heat Transfer are solved by employing a regular perturbation technique the velocity and temperature distributions are analysed for different values of the Hartmann Number M , Hallcurrent parameter m , Grashoff Number G , and Radiation parameter N_1 . The rate of Heat Transfer has been calculated numerically for different values of the governing parameters.

Keywords: Heat Transfer, Hall effects, Heat sources, Vertical channel.

1. INTRODUCTION

The magneto hydrodynamic heat transfer has gained significance in recent times owing to its applications in recent advancement of space technology. The process of free convection as a mode of heat transfer has wide applications in the fields of chemical Engineering, Aeronautics and Nuclear power generation. Natural conve-

ction in porous media with internal heat generation is of interest in such situations as post accident heat removal in nuclear power reactors and the geothermal problems arising during the storage of nuclear waste in the earth². Rajeswara Rao⁶ has analysed the combined free and forced convective flow of an electrically conducting, viscous, incompressible fluid confined in a vertical channel whose boundaries are maintained at

a non-uniform temperature He has not considered induced magnetic field into account Taking induced magnetic field into account Ram Chandra⁷ has discussed the free and forced convective flow of an electrically conducting fluid in a vertical channel whose walls are maintained at non-uniform temperature Ravindra⁸ has investigated the natural convective flow and heat transfer through a porous medium in a vertical channel maintained at Non-uniform temperature with constant heat sources.

The unsteady flow of a rotating viscous fluid in a channel maintained by non-tensional oscillations of one or both the boundaries has been studied by several authors to analyse the growth and development of boundary layers associated with geothermal flows for possible applications in geophysical fluid dynamics^{2,3,4,5,9}. Later Singh *et al.*¹⁰ studied free convection in MHD flow of a rotating viscous liquid in porous media. Singh *et al.*¹¹ have also studied free convective MHD flow of a rotating viscous fluid in a porous medium past an infinite vertical porous plate. However, in a partially ionized gas, there occurs a Hall current when the strength of the impressed magnetic field is very strong.

2. FORMULATION

We consider the steady flow of an incompressible, viscous, electrically conducting fluid confined in a vertical channel bounded by two flat walls under the influence of an inclined magnetic field of intensity H_0 lying in the plane (y-z).The

magnetic field is inclined at an angle α to the axial direction k and hence its components are $(0, H_0 \sin(\alpha), H_0 \cos(\alpha))$. The walls are maintained at non-uniform temperature. In view of the non-uniform boundary temperature the velocity field has components $(u, 0, w)$. The magnetic field in the presence of fluid flow induces the current $(J_x, 0, J_z)$. We choose a rectangular cartesian co-ordinate system $O(x, y, z)$ with z-axis in the vertical direction and the walls at $x = \pm L$.

When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm's law is modified to

$$\bar{J} + \omega_e \tau_e \bar{J} \times \bar{H} = \sigma (\bar{E} + \mu_e \bar{q} \times \bar{H}) \quad (1)$$

where q is the velocity vector. H is the magnetic field intensity vector. E is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time, σ is the fluid conductivity and μ_e is the magnetic permeability.

Neglecting the electron pressure gradient, ion-slip and thermo-electric effects and assuming the electric field $E=0$, equation (2.6) reduces

$$J_x - m H_0 J_z \sin(\alpha) = -\sigma \mu_e H_0 w \sin(\alpha) \quad (2)$$

$$J_z + m H_0 J_x \sin(\alpha) = \sigma \mu_e H_0 u \sin(\alpha) \quad (3)$$

where $m = \omega_e \tau_e$ is the Hall parameter.

On solving equations (2) & (3) we obtain

$$J_x = \left(\frac{\sigma \mu_e H_0 \sin(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} \right) (m H_0 \sin(\alpha) - w) \quad (4)$$

$$J_z = \left(\frac{\sigma \mu_e H_0 \sin(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} \right) (u + m H_0 w \sin(\alpha)) \quad (5)$$

where u, w are the velocity components along x and z directions respectively, On introducing the following non-dimensional variables

$$(x', z') = \frac{(x, z)}{L}, \psi' = \frac{\psi}{qL}, \theta = \frac{T - T_e}{\Delta T_e}$$

The equations of momentum and energy in the presence of heat generating sources in the non-dimensional form are

$$\nabla^4 \psi - M_1^2 \nabla^2 \psi + \left(\left(\frac{G}{R} \right) \left(\frac{\partial \theta}{\partial x} \right) \right) = R \left(\left(\frac{\partial \psi}{\partial z} \right) \left(\frac{\partial (\nabla^2 \psi)}{\partial x} \right) - \left(\frac{\partial \psi}{\partial x} \right) \left(\frac{\partial (\nabla^2 \psi)}{\partial z} \right) \right) \quad (6)$$

$$PR \left(\left(\frac{\partial \psi}{\partial x} \right) \left(\frac{\partial \theta}{\partial z} \right) - \left(\frac{\partial \psi}{\partial z} \right) \left(\frac{\partial \theta}{\partial x} \right) \right) = \nabla^2 \theta - \alpha \theta \quad (7)$$

where

$$G = \left(\frac{\beta g \Delta T_e L^3}{\nu^2} \right) \quad (\text{Grashof Number})$$

$$M^2 = \left(\frac{\sigma \mu_e^2 H_0^2 L^2}{\nu^2} \right) \quad (\text{Hartman Number})$$

$$R = \left(\frac{qL}{\nu} \right) \quad (\text{Reynolds Number})$$

$$P = \left(\frac{\mu C_p}{K_f} \right) \quad (\text{Prandtl Number})$$

$$\alpha = \left(\frac{QL^2}{K_f} \right) \quad (\text{Heat Source Parameter})$$

The corresponding boundary conditions are

$$\psi(f) - \psi(-f) = 1$$

$$\frac{\partial \psi}{\partial z} = 0, \frac{\partial \psi}{\partial x} = 0, \theta = \gamma(\delta z) \quad \text{at } x = -L$$

$$\frac{\partial \psi}{\partial z} = 0, \frac{\partial \psi}{\partial x} = 0, \theta = \gamma(\delta z) \quad \text{at } x = +L$$

3. METHOD OF SOLUTION

Introduce the transformation such that $\bar{z} = \delta z$, $\frac{\partial}{\partial z} = \delta \frac{\partial}{\partial \bar{z}}$

$$\text{Then } \frac{\partial}{\partial z} \approx O(\delta) \rightarrow \frac{\partial}{\partial \bar{z}} \approx O(1)$$

For small values of $\delta \ll 1$, the flow develops slowly with axial gradient of order δ and hence we take $\frac{\partial}{\partial \bar{z}} \approx O(1)$.

Using the above transformation the equations governing equations reduces to

$$F^4 \psi - M_1^2 F^2 \psi + \left(\left(\frac{G}{R} \right) \left(\frac{\partial \theta}{\partial x} \right) \right) = \delta R \left(\frac{\partial \psi}{\partial \bar{z}} \frac{\partial (F^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial (F^2 \psi)}{\partial \bar{z}} \right) \quad (8)$$

$$\delta PR \left(\left(\frac{\partial \psi}{\partial x} \right) \left(\frac{\partial \theta}{\partial \bar{z}} \right) - \left(\frac{\partial \psi}{\partial \bar{z}} \right) \left(\frac{\partial \theta}{\partial x} \right) \right) = F^2 \theta - \alpha \theta \quad (9)$$

where

$$F^2 = \frac{\partial}{\partial x^2} + \delta^2 \frac{\partial}{\partial \bar{z}^2} \quad \left(\frac{\partial^4 \psi_1}{\partial y^4} - (M_1^2) \frac{\partial^2 \psi_1}{\partial y^2} \right) = - \left(\left(\frac{G}{R} \right) \left(\frac{\partial \theta_1}{\partial \bar{z}} \right) \right) + R \left(\left(\frac{\partial \psi_0}{\partial y} \right) \left(\frac{\partial^3 \psi_0}{\partial z^3} \right) - \left(\frac{\partial \psi_0}{\partial \bar{z}} \right) \left(\frac{\partial^3 \psi_0}{\partial x \partial y^2} \right) \right) \quad (15)$$

Assuming the slope δ of the wavy boundary to be small we take

$$\begin{aligned} \psi(x, z) &= \psi_0(x, y) + \delta \psi_1(x, z) + \delta^2 \psi_2(x, z) + \dots \\ \theta(x, z) &= \theta_0(x, z) + \delta \theta_1(x, z) + \delta^2 \theta_2(x, z) + \dots \end{aligned} \quad (10)$$

Substituting (10) in equations (8)&(9) and equating the like powers of δ the equations and the respective boundary conditions to the zeroth order are

$$\frac{\partial^2 \theta_0}{\partial y^2} - \alpha_1 \theta_0 = 0 \quad (11)$$

$$\left(\frac{\partial^4 \psi_0}{\partial y^4} - (M_1^2) \frac{\partial^2 \psi_0}{\partial y^2} \right) = - \left(\left(\frac{G}{R} \right) \left(\frac{\partial \theta_0}{\partial \bar{z}} \right) \right) \quad (12)$$

with

$$\begin{aligned} \psi_0(+1) - \psi_0(-1) &= 1 \\ \frac{\partial \psi_0}{\partial y} = 0, \frac{\partial \psi_0}{\partial \bar{z}} = 0, \theta_0 = \gamma(\bar{z}) &\text{ at } x = -1 \\ \frac{\partial \psi_0}{\partial y} = 0, \frac{\partial \psi_0}{\partial \bar{z}} = 0, \theta_0 = \gamma(\bar{z}) &\text{ at } x = +1 \end{aligned} \quad (13)$$

and to the first order are

$$\left(\frac{\partial^2 \theta_1}{\partial y^2} - \alpha \theta_1 \right) = P R \quad \left(\left(\frac{\partial \psi_0}{\partial x} \right) \left(\frac{\partial \theta_0}{\partial \bar{z}} \right) - \left(\frac{\partial \psi_0}{\partial \bar{z}} \right) \left(\frac{\partial \theta_0}{\partial y} \right) \right) \quad (14)$$

with

$$\begin{aligned} \psi_1(+1) - \psi_1(-1) &= 0 \\ \frac{\partial \psi_1}{\partial y} = 0, \frac{\partial \psi_1}{\partial \bar{z}} = 0, \theta_1 = 0 &\text{ at } x = -1 \\ \frac{\partial \psi_1}{\partial y} = 0, \frac{\partial \psi_1}{\partial \bar{z}} = 0, \theta_1 = 0 &\text{ at } x = +1 \end{aligned} \quad (16)$$

Where

$$N_2 = \frac{3N_1}{3N_1 + 4}, \alpha_1 = \alpha N_2, P_1 = P N_2$$

4. RATE OF HEAT TRANSFER

The rate of Heat transfer (Nusselt Number) on the walls has been calculated using the formula

$$Nu = \frac{1}{(\theta_m - \theta_w)} \left(\frac{\partial \theta}{\partial y} \right)_{y=\pm 1}$$

$$\text{where } \theta_m = 0.5 \int_{-1}^1 \theta dy$$

and the corresponding expressions are

$$(Nu)_{y=-1} = \frac{1}{(\theta_m - 1)} (-a_{92} + \delta a_{94})$$

$$(Nu)_{y=+1} = \frac{1}{\theta_m} (a_{92} + \delta a_{93})$$

5. ANALYSIS OF THE NUMERICAL RESULTS

In this analysis we investigate the effect of Hall currents on Convective Heat Transfer flow of a Viscous Incompressible electrically conducting fluid in a Non-Uniformly vertical channel. We take the Prandtl Number $P=0.71$ and $\delta = 0.01$. The Velocity components u, w and The Non-dimensional Temperature distribution θ are shown in Figs.1-6. For different values of M, m , and N_1

The Secondary velocity (u) which is due to the Non-uniform Temperature on the boundary is depicted in Figs.1 and 2 for different parametric values. It is noticed from Fig.1 represents the variation of u with M and m it is found that the secondary velocity retards with increase in Hall parameter m while it enhances with increase in the Hall parameter m in the entire flow region.

Fig.2 shows that the variation of u with Radiation Parameter N_1 shows that an increase in N_1 results in an enhancement in $|u|$.

The axial velocity w is shown in Figs.3 and 4. The variation of w with Hartmann number M and Hall parameter m is exhibited in Fig.3. It is observed that higher the Lorentz force smaller $|w|$ in the flow region also $|w|$ enhances with increase in the Hall parameter m . The effect of radiation parameter N_1 on the axial flow w is

shown in Fig.4. It is found that an increase in $N_1 \leq 5$, $|w|$ depreciates in the left region and enhances in the right region and for higher $N_1 \geq 10$, $|w|$ depreciates in the flow region except in the narrow region adjacent the lower boundary $y=-1$ and this region reduces with increase in N_1 .

The Non-dimensional Temperature distribution (θ) is exhibited in Figs.5 and 6 for different values of M, m , and N_1 . It is found that the temperature is positive for all parametric values variation. The variation of θ with M and m shows that higher the Lorentz force larger the actual temperature and for further increase in the Lorentz force smaller the actual temperature in the entire flow region. An increase in the Hall parameter $m \leq 1.5$ depreciates the actual temperature and for further higher $m \geq 2.5$. We notice an enhancement in the vicinity of the boundary $y = \pm 1$ and depreciates marginally in the central region of the flow (Fig.5). The variation of θ with Radiation parameter N_1 . It is found that θ reduces with increase in N_1 the inclusion of the radiative Heat Transfer leads to the remarkable depreciation in the Non-dimensional Temperature θ (Fig.6).

The average Nusselt Number (Nu) which represents the which measures the rate of Heat Transfer at $y = \pm 1$ is shown in Tables.1-4 for different variations of G, M, m, α , and R . It is found that the rate of Heat Transfer depreciates with increase in the $G > 0$ and enhances with $G < 0$ at both the boundaries. The variation of Nu with Hartmann Number M shows that the higher the Lorentz force larger $|Nu|$ at $y = +1$ and

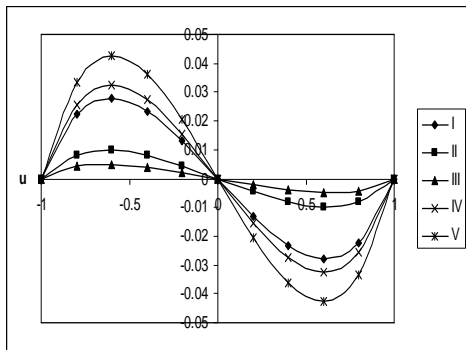


Fig. 1 : u with M & m

	I	II	III	IV	V
M	10	15	20	10	10
m	0.5	0.5	0.5	1.5	2.5

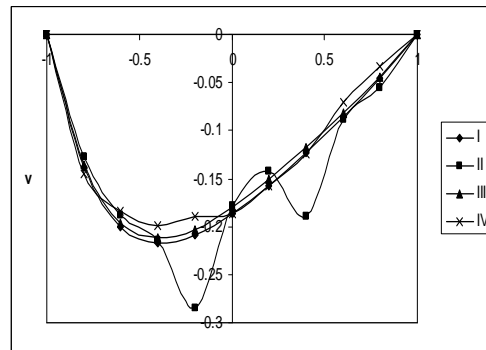


Fig. 4 : w with N_1

	I	II	III	IV
N_1	2.5	5	10	100

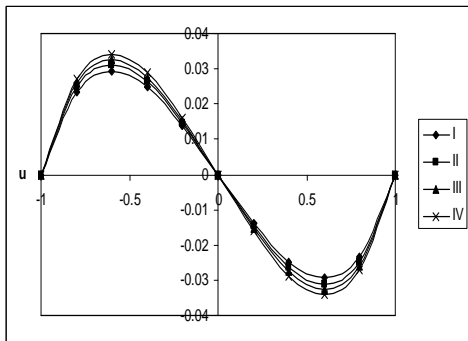


Fig. 2 : u with N_1

	I	II	III	IV
N_1	2.5	5	10	100

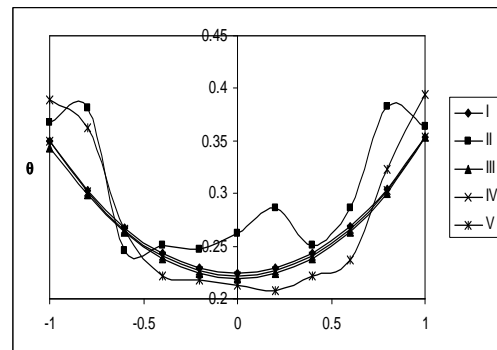


Fig. 5 : θ with M & m

	I	II	III	IV	V
M	10	15	20	10	10
m	0.5	0.5	0.5	1.5	2.5

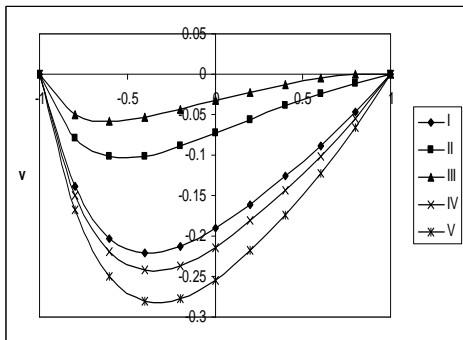


Fig. 3 : w with M & m

	I	II	III	IV	V
M	10	15	20	10	10
m	0.5	0.5	0.5	1.5	2.5

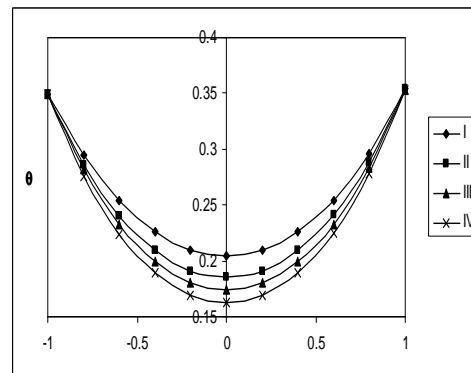


Fig. 6 : θ with N_1

	I	II	III	IV
N_1	2.5	5	10	100

smaller $|Nu|$ at $y=-1$ in the heating case while in the cooling case smaller $|Nu|$ for $M \leq 15$ and larger $|Nu|$ with $M \geq 20$ at $y=+1$ and at $y=-1$ smaller $|Nu|$ an increase in the

Hall parameter m enhances $|Nu|$ in the heating case and in the cooling case it reduces with $m \leq 1.5$ and enhances with $m \geq 2.5$. Also it enhances with increase in the Heat source parameter α and Reynolds Number R . (Tables.1 and 2).

Table . 1 Average Nusselt Number (Nu) at $y=+1$ $P=0.71$, $x=\pi/4$, $\alpha_1=0.3$, $N_1=1.5$

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	0.5514	0.5822	0.7438	0.6635	0.6915	2.3829	-20.9774	0.6228	0.7665
2×10^3	0.5097	0.5593	0.7272	0.7312	0.7180	2.4844	-19.5531	0.6015	0.7561
-10^3	0.6364	0.6288	0.7772	0.5379	0.6405	2.2118	13.4060	0.6657	0.7872
-2×10^3	0.6797	0.6524	0.7941	0.4797	0.6161	2.1390	8.7229	0.6872	0.7974

Table . 2 Average Nusselt Number (Nu) at $y=-1$ $p=0.71$, $x= \pi /4$, $\alpha_1=0.3$, $N_1=1.5$

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-0.4734	-0.3979	-0.2447	-0.5883	-0.6207	-0.21886	13.5318	-0.5378	-0.6635
2×10^3	-0.4323	-0.3757	-0.2295	-0.6541	-0.6463	-2.2773	18.3357	-0.5164	-0.6528
-10^3	-0.5574	-0.4428	-0.2753	-0.4664	-0.5718	-2.0373	-12.5995	-0.5805	-0.6848
-2×10^3	-0.6002	-0.4656	-0.2908	-0.4099	-0.5484	-1.9732	-8.2040	-0.6019	-0.6953

M	10	15	20	10	10	10	10	10	10
m	0.5	0.5	0.5	1.5	2.5	0.5	0.5	0.5	0.5
α	2	2	2	2	2	4	6	2	2
R	35	35	35	35	35	35	35	70	140

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