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The Onset of Convection in Walters' (Model B') fluid in a Darcy-Brinkman Porous Medium

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ABSTRACT

The onset of convection in Walters' (Model B') elastico-viscous fluid in porous medium heated from below is considered. For the porous medium, the Brinkman model is employed. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the medium permeability, gravity field and viscoelasticity introduce oscillatory modes. For stationary convection, the Darcy number has stabilizing effect whereas the medium permeability has destabilizing effect on the system. The effects of medium permeability and Darcy number has also been shown graphically.

Keywords: Walters' (Model B') fluid, thermal convection, viscoelasticity, porous medium.

1. INTRODUCTION

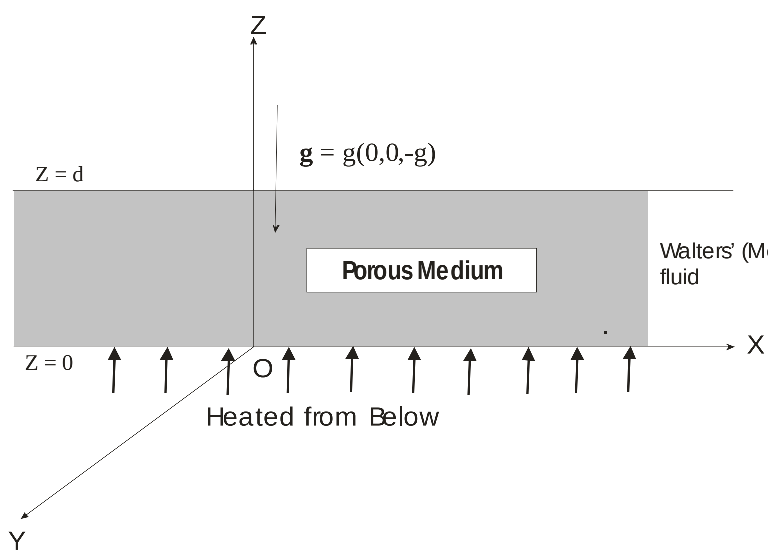
In recent years, considerable interest has been evinced in the study of thermal instability in a porous medium, because it has various applications in geophysics, food processing and nuclear reactors. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar[1]. Lapwood[4] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding[11].

There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walters' (Model B') elastico-viscous fluid having relevance and in chemical technology and industry. Walters[10] reported that the mixture of polymethyl methacrylate and pyridine at 25^oC containg 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters' (Model B') elastico-viscous fluid. Walters' (Model B') elastico-viscous fluid form the basis for the manufacture of many important polymers and useful products. A good account of convection problems in a porous medium is given by Vafai and Hadim[9], Ingham and Pop[2] and Nield and Bejan[5].

The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. Kuznetsov and Nield[3] have studied thermal instability in a porous medium layer saturated by a nanofluid: Brinkman model. Sharma and Rana[8] have studied thermal instability of a Walters' (Model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium. Recently, Rana and Kango[6] have been studied the effect of rotation on thermal instability of Compressible Walters' (Model B') elastico-viscous fluid in porous medium Rana and Kango[7] have also studied thermal instability of compressible Walters' (Model B') elastico-viscous rotating fluid permitted with suspended dust particles in porous medium

The interest for investigations of non-Newtonian fluids is also motivated by a wide range of engineering applications which include ground pollutions by chemicals which are non-Newtonian like lubricants and polymers and in the treatment of sewage sludge in drying beds. Recently, polymers are used in agriculture, communications appliances and in bio medical applications. Examples of these applications are filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc.

Keeping in mind the importance in various applications mentioned above, our interest, in the present paper is to study the onset of convection in Walters' (Model B') fluid in a Brinkman porous Medium.



Schematic Sketch of Physical Situation

2. MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Here we consider an infinite, horizontal, incompressible Walters' (Model B') elastico-viscous fluid of depth d, bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by gravity g(0, 0, -g). This layer is heated from below such that a steady adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

Let $\rho, \upsilon, \upsilon', p, \in$, T, $\tilde{\mu}$, α and $\boldsymbol{v}$(0, 0, 0), denote, respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, effective viscosity, thermal coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass, temperature and equation of state for Walters' (Model B') elastico-viscous fluid are

$$\frac{1}{\in}\left[\frac{\partial q}{\partial t}+\frac{1}{\varepsilon}(q.\nabla)\boldsymbol{v}\right]=-\frac{1}{\rho_0}\nabla p+g\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{v}+\frac{\tilde{\mu}}{\rho_0}\nabla^2\boldsymbol{v}, \tag{1}$$

$$\nabla.\boldsymbol{v}=0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+(v.\nabla)T=\kappa\nabla^2 \mathrm{T}, \tag{3}$$

$$\rho=\rho_0[1-\alpha(T-T_0)], \tag{4}$$

where the suffix zero refers to values at the reference level z = 0.

Here

$$E=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$$

which is constant, κ is the thermal diffusivity, ρ_s, c_s; ρ_0, c_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$\boldsymbol{v}=(0,0,0),\ T=-\beta z+T_0,$$
$$\rho=\rho_0(1+\alpha\beta z), \tag{5}$$

is an exact solution to the governing equations.

Let $\boldsymbol{v}$(u,v,w), $\boldsymbol{v_d}$(l,r,s), θ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $\boldsymbol{v}$(0,0,0), the perturbation in particle velocity $\boldsymbol{v_d}$(0,0,0), temperature T, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation θ temperature is given by

$$\delta\rho=-\alpha\rho_0\theta. \tag{6}$$

The linearized perturbation equations governing the motion of fluids are

$$\frac{1}{\varepsilon}\frac{\partial\boldsymbol{v}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-g\frac{\delta\rho}{\rho_0}-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{v}+\frac{\tilde{\mu}}{\rho_0}\nabla^2\boldsymbol{v}, \tag{7}$$

$$\nabla.\boldsymbol{v}=0, \tag{8}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\boldsymbol{v}_d=\boldsymbol{v}, \tag{9}$$

$$E\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta, \tag{10}$$

where w is the vertical fluid velocity.

In the Cartesian form, equations (7)-(10) with the help of equation (6) can be expressed as

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial x}(\delta p) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)u + \frac{\tilde{\mu}}{\rho_0}\nabla^2 u, \quad (11)$$

$$\frac{1}{\varepsilon}\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial y}(\delta p) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)v + \frac{\tilde{\mu}}{\rho_0}\nabla^2 v, \quad (12)$$

$$\frac{1}{\varepsilon}\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}(\delta p) - g\alpha\theta - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)w + \frac{\tilde{\mu}}{\rho_0}\nabla^2 w\,, \quad (13)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (14)$$

$$E\frac{\partial \theta}{\partial t} = \beta w + \kappa\nabla^2\theta. \quad (15)$$

Operating equation (11) and (12) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using equation (14), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) = \frac{1}{\rho_0}\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\delta p - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial z}\right) + \frac{\tilde{\mu}}{\rho_0}\nabla^2\left(\frac{\partial w}{\partial z}\right), \quad (16)$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity.

Operating equation (13) and (16) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate δp between equations (13) and (16), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 \mathrm{w}) = -\frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\nabla^2 \mathrm{w} + \frac{\tilde{\mu}}{\rho_0}\nabla^4 \mathrm{w} + \mathrm{g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\alpha\theta, \quad (17)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

3. THE DISPERSION RELATION

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, s, \theta, \zeta] = [W(z), S(z), \Theta(z), Z(z)]exp(ilx + imy + nt), \quad (18)$$

where *l* and *m* are the wave numbers in the x and y directions, $k = (l^2 + m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.
Using expression (18) in equations (17) and (15) become

$$\frac{n}{\varepsilon}\left[\frac{d^2}{dz^2} - k^2\right]W = -gk^2\alpha\Theta - \frac{1}{k_1}(\upsilon - \upsilon' n)\left(\frac{d^2}{dz^2} - k^2\right)W + \frac{\tilde{\mu}}{\rho_0}\left(\frac{d^2}{dz^2} - k^2\right)^2 W, \quad (19)$$

$$En\Theta = \beta W + \kappa\left(\frac{d^2}{dz^2} - k^2\right)\Theta. \quad (20)$$

Equation (19) and (20) in non dimensional form, become

$$\left[\frac{\sigma}{\epsilon} + \frac{1 - F\sigma}{P_l}\right](D^2 - a^2)W + \frac{ga^2 d^2\alpha\Theta}{\upsilon} + \frac{\tilde{\mu}}{\mu}(D^2 - a^2)^2 W = 0, \quad (21)$$

$$[(D^2 - a^2) - Ep_1\sigma]\Theta = -\frac{\beta d^2}{\kappa}W, \quad (22)$$

where we have put

$a = kd$, $\sigma = \frac{nd^2}{\upsilon}$, $F = \frac{\upsilon'}{d^2}$ and $P_l = \frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1 = \frac{\upsilon}{\kappa}$, is the thermal Prandtl number.

Eliminating Θ between equations (22) and (21), we obtain

$$\left[1 + \left(\frac{P_l}{\varepsilon} - F\right)\sigma\right](D^2 - a^2)(D^2 - a^2 - Ep_1\sigma)W - Ra^2 P_l W + D_A(D^2 - a^2)^2(D^2 - a^2 - E_1 p_1\sigma)W = 0, \tag{23}$$

where $R = \frac{g\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number and $D_A = \frac{\tilde{\mu}k_1}{\mu d^2}$, is the Brinkman-Darcy number modified by the viscosity ratio.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar[1])

$$W = D^2W = D^3 W = \Theta = 0 \quad \text{at } z = 0 \text{ and } 1. \tag{24}$$

The case of two free boundaries, though a little artificial is the most appropriate for stellar atmospheres. Using the boundary conditions (24), we can show that all the even order derivatives of W must vanish for z = 0 and z = 1 and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z\,; \quad W_0 \text{ is a constant.} \tag{25}$$

Substituting equation (24) in (22), we obtain the dispersion relation

$$R_1 xP = \left[1 + \left(\frac{P}{\varepsilon} - \pi^2 F\right) i\sigma_1\right](1 + x)(1 + x + Ep_1 i\sigma_1) + D_{A_1}(1 + x)^2(1 + x + E_1 p_1 i\sigma_1), \tag{26}$$

where $R_1 = \frac{R}{\pi^4}$, $D_{A_1} = \frac{D_A}{\pi^4}$, $x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $P = \pi^2 P_l$.

Equation (26) is required dispersion relation accounting for the onset of thermal convection in Walters' (Model B') elastico-viscous fluid in a Brinkman porous medium.

4. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Here we examine the possibility of oscillatory modes, if any, in Walters (Model B') elastico-viscous fluid due to the presence of viscoelasticity, medium perpeability and gravity field. Multiply equation (21) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (22) with the help of boundary conditions (24), we obtain

$$\left[1 + \left(\frac{P_l}{\varepsilon} - F\right)\sigma\right] I_1 - \frac{ga^2\alpha\kappa P_l}{\upsilon\beta}(I_2 + Ep_1\sigma^* I_3) - D_A(I_4 + a^2 I_5) = 0, \tag{27}$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)\,dz,$$
$$I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)\,dz,$$
$$I_3 = \int_0^1 |\Theta|^2 dz,$$
$$I_4 = \int_0^1 |DW|^4 dz,$$
$$I_5 = \int_0^1 (2|DW|^2 + a^2|W|^2)\,dz.$$

The integral part I_1-I_5 are all positive definite. Putting $\sigma = i\sigma_i$ in equation (26), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left[\left(\frac{P_l}{\varepsilon}-F\right)I_1+\frac{ga^2\alpha\kappa P_l}{\upsilon\beta}Ep_1I_3\right]=0\,, \qquad (28)$$

Equation (28) implies that $\sigma_i=0$ or $\sigma_i\neq 0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes introduced due to presence of viscoelasticity, gravity field and medium permeability.

5. THE STATIONARY CONVECTION

For stationary convection putting $\sigma=0$ in equation (25) reduces it to

$$R_1=\frac{(1+x)^2}{xP}\left[1+(1+x)D_{A_1}\right] \qquad (29)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters D_{A_1}, P and Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ .

To study the effects of Darcy number and medium permeability, we examine the behavior of $\frac{dR_1}{dD_{A_1}}$ and $\frac{dR_1}{dP}$ analytically.

From equation (29), we get

$$\frac{dR_1}{dD_{A_1}}=\frac{(1+x)^3}{xP}\,, \qquad (30)$$

which shows that Darcy number has stabilizing effect on the system.

It is evident from equation (29) that

$$\frac{dR_1}{dP}=-\frac{(1+x)^2}{xP^2}\left[1+(1+x)D_{A_1}\right]\,, \qquad (31)$$

From equation (30), we observe that medium permeability has destabilizing effect.

The dispersion relation (29) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

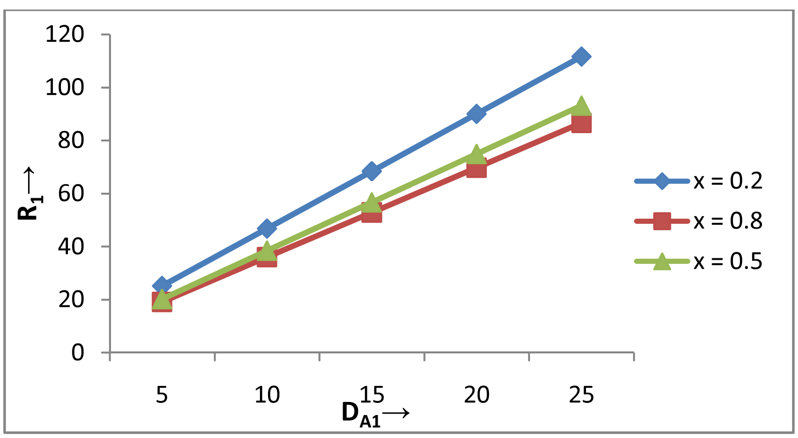


Fig.1. Variation of Rayleigh number R_1 with Darcy number D_{A_1} for P = 2 for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

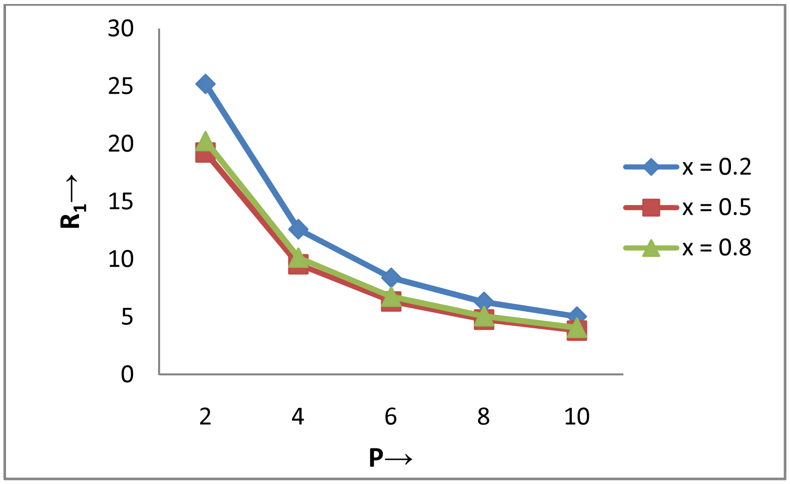


Fig.2. Variation of Rayleigh number R_1 with medium permeability P for $D_{A_1} = 5$ for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

In fig.1, Rayleigh number R_1 is plotted against rotation D_{A_1} for B = 3, P = 2 for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8. This shows that Darcy number has a stabilizing effect for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

In fig.2, Rayleigh number R_1 is plotted against medium permeability P for $D_{A_1} = 5$ for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8. This shows that medium permeability has a destabilizing effect.

6. CONCLUSION

The onset of thermal convection in Walters' (Model B') elastico-viscous fluid in a Darcy-Brinkman porous medium has been investigated. The dispersion relation, including the effects of Darcy number, medium permeability and viscoelasticity on the thermal instability in Walters' (Model B') fluid is derived. From the analysis, the main conclusions are as follows:

(i) For the case of stationary convection, Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid as elastico-viscous parameter F vanishes with σ .

(ii) The expressions for $\frac{dR_1}{dD_{A_1}}$ and $\frac{dR_1}{dP}$ are examined analytically and it has been found that the Darcy number has stabilizing effect whereas the medium permeability has a destabilizing effect on the system. The effects of Darcy number and medium permeability on thermal convection have also been shown graphically in figures 1 and 2.

(iii) The oscillatory modes introduced due to presence of viscoelasticity, gravity field, and medium permeability.

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