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A Theoretical Study of Super Fluidity in Spin-orbit Coupled Bose-Einstein Condensate

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ABSTRACT

Using the theoretical formalism of Q Zhu *et al*. [EPL, 109, 50003 92012) and Y. J. Lin *et al*.[Nature 471, 83 (2011)], we have theoretically studied spin-orbit coupling and super fluidity in SOC coupled Bose Einstein condensate. In this study, we observed the following facts:

Spin-orbit coupling plays an essential role in the super fluidity of SOC coupled Bose Einstein condensate.

Our theoretical analysis of the Bogoliubov excitation shows that it has two branches (a) one is gapless and phonon like at long wavelength (b) typically gapped.

The excitation implies super fluidity which contains distinct new features: (i) Galilean invariance is absent (ii) One cannot define critical velocity of super fluidity independent of the reference frame. The super fluidity depends upon two factors (a) the speed of BEC exceeds a critical value (b) cross-helicity. The cross helicity is defined as the cross product of the spin and kinetic momentum of the BEC.

Keywords: Spin-orbit coupling, Spin-orbit coupled super fluidity, Topological insulators, Spin Hall effect, Spintronic devices, Rashba spin interaction term, Dresselhaus spin interaction term, Bogoliubov excitation, critical velocity, Cross-helicity.

INTRODUCTION

Super fluidity was first discovered in 1938 and has fascinated physicists ever since. This interesting phenomenon was explained by Landau[1], whose theory has been very successful in explaining many important properties of super fluids. However, Landau's theory of super fluidity may be facing challenges brought by the recent experimental realization of artificial gauge fields for ultra-cold bosonic atoms[2-5]. When the artificial gauge field is nonAbelian[6-8], it is effectively spin-orbit coupling (SOC). SOC has played a crucial role in many exotic phenomena such as spin Hall effect[9] and topological insulators[10]. SOC is the interaction between a quantum particle spin and its momentum. It contributes to the electronic properties of materials such as GaAs and is important for spintronic devices[11]. Quantum many body systems of ultra-cold atoms can be precisely controlled experimentally and can provide an ideal platform on which one can study SO coupling. Although an atom's intrinsic SO coupling affects its electronic structure, it does not lead to coupling between the spin and centre- of mass notion of the atm. In neutral Bose Einstein condensate, one considers SO coupling with equal Rashba[12] and Dresselhaus[13] strength by dressing two atomic spin states with pair of lasers. Such coupling has not been realized previously for ultracold atomic gases or any bosonic system. Furthermore, in the presence of laser coupling, the interactions between the two dressed atomic spin states are modified. This drives a quantum phase transition from a spatially spin mixed states to a phase oriented states. One develops a many-body theory that provides quantitative agreement with the observed location of the transition. The SO coupling is equally applicable to bosons and fermions. It sets the stage for the realization of topological insulators in fermionic neutral atom systems.

There have been some theoretical works, where many interesting properties of spin-orbit coupled BEC are explored[14]. SOC can lead to unconventional BEC with the breaking of time reversal symmetry. Later on a stripe phase that breaks the rotational symmetry was found.

MATHEMATICAL FORMULA USED IN THE STUDY

Our study of the super fluidity is based on the computation of the elementary excitations using the Bogoliubov equation. One calculates how the elementary excitations change with flow speed and manage to derive from these excitations for the critical speed from the two different cases. One finds that there are two branches of elementary excitations for a BEC with SOC. The lower branch is phonon-like at long wavelength and the upper branch is generally gapped. Careful analysis of these excitations indicates that the critical velocity for a BEC with SOC is non-zero while the critical dragging speed is zero. This shows that the critical velocity depends on the reference frame for a BEC with a SOC and probably for any super fluid that has no Galilean invariance. In addition, one finds that the properties of a flow of BEC with SOC are also related to its spin direction. One characterizes this spin direction with cross-helicity which is defined as the cross-product of the spin and kinetic momentum of the flow. A BEC with Rashaba SOC is always unstable if its cross-helicity is negative.

One considers a BEC with pseudospin ½ and Rashba SOC. The system can be described by the Hamiltonian[15]

$$H = \int dr\{\Sigma_{\sigma=1,2}\psi^{\dagger}{}_{\sigma}(\frac{-\hbar^2\nabla^2}{2m}+V(r))\psi_{\sigma}+\gamma[\psi^{*}{}_{1}(ip_x+p_y)\psi_2+\psi^{*}{}_{2}(-ip_x+p_y)\psi_1]+(C_1/2)$$

$$(|\psi_1|^4 + |\psi_2|^4) + C_2 |\psi_1|^2 |\psi_2|^2\} \quad (1)$$

Here, γ is the SOC constant, C_1 and C_2 are interaction strength between the same and different pseudospin states respectively. One considers the homogeneous case $V(r) = 0$ and $C_1>C_2$ when the system is stable against the phase separation. One also considers that BEC moves in y-direction and the critical velocity is found to be not influenced by the excitation in the z-direction.

The Gross-Pitaevskii equation obtained from the Hamiltonian (1) has plane wave solutions

$$\Phi_{k^\rightarrow} = (\psi_1) = \frac{1}{\sqrt{2}} \ e^{i\vartheta_k} \ e^{ik^\rightarrow \cdot r^\rightarrow - i\mu(k^\rightarrow)t}$$

$$(\psi_2) = \frac{1}{\sqrt{2}} \{-1\} e^{ik^\rightarrow \cdot r^\rightarrow - i\mu(k^\rightarrow)t} \quad (2)$$

Here, $\tan\vartheta_k = \frac{k_x}{k_y}$, $\mu(k^\rightarrow) = (|k^\rightarrow|^2 / 2) - \gamma|k^\rightarrow| + (C_1 + C_2)/2$. The solution $\vartheta_{k^\rightarrow}$ is the ground state of the system when $|k^\rightarrow| = \gamma$. The plane wave solution $\vartheta_{k^\rightarrow}$ represents a BEC flow with the velocity $v^\rightarrow = k^\rightarrow - \gamma k$. Here $k^\rightarrow = k\hat{y}$.

DETERMINATION OF CRITICAL VELOCITIES

Now, one considers the case, where the BEC flows with a given velocity. The two branches of excitations are given by

$$\varepsilon_\pm(q^\rightarrow) = q_y k + [((C_1 \pm C_2)/2)\mathrm{q}^2 + (q^2/4)]^{\frac{1}{2}} \quad (3)$$

These results show that the system at the ground state (k=0) has two different speed of sound, $\sqrt{(C_1 + C_2)/2}$ and $\sqrt{(C_1 - C_2)/2,}$. Since the excitation ε_- becomes negative only when k> $\sqrt{(C_1 + C_2)/2}$, the critical flowing velocity in this case is $\sqrt{(C_1 - C_2)/2,}$. When $C_2 = 0$, these two branches of excitatios merge into one and the critical velocity is $\sqrt{\frac{C_1}{2}}$.This is well known result and was confirmed in BEC experiment[16].

DISCUSSION OF RESULTS

Using the theoretical formalism of Q Zhu *et al*. and Y. J. Lin *et al*., we have theoretically studied super fluidity in spin-orbit coupled Bose-Einstein condensate. The studies have been performed by computing its Bogoliubov excitations. We observed that the excitations have two branches. One is gapless phonon like at long wavelength and the other is typically gapped. These excitations imply super fluidity that has new features. Here the critical velocity for the super fluidity shows that it exceeds a critical value but also on cross-helicity which is defined as the cross-product of the spin and kinetic momentum of the BEC. In **table T1**, we have shown the evaluated results of the excitation energy as a function of wave vector q_x along x-axis with different values of conjugate momentum k. The evaluation is done for three values of k namely k=1, k=2.5 and k=4. Our evaluated values show that excitation energy decrease as function of q_x for all the values of k taken. The result also indicate that the excitation energy become zero for k=1 and for other two values of k, it shows some minimum value. In **table T2,** we repeated the similar calculation in y direction. Here excitation energy is evaluated as a function of q_y. for fixed value of k. We have taken the same value of k in this calculation also. Here, we observed that the excitation energy decrease with q_y in all the three cases. The results indicate that here the excitation energy decrease and attain a minimum value and then increases for each case of k. For k=1, the minimum value is 1.6, for k=2.5., the minimum is 0.6 and for k=4, no minimum is obtained, the value increase from very beginning to the end. The results also indicate that for lower value of k it has clearly two branches but as one increases the value of k the branch separation is vanished. In t**able T3**, we have shown the evaluated results of the critical velocity v_c as a function of SOC parameter γ keeping interaction strength $C_1>3C_2$. Our obtained result shows that velocity increases with γ and becomes flat. We obtained that the values of $C_1=11$ and $C_2=4$ the flatness begins. In **table T4,** we repeated the calculation of the critical velocity as a function of γ with the condition that $C_1<3C_2$. Here again we found the similar behavior at $C_1=14$ and $C_2=3$. Our theoretically obtained results are in good agreement with other theoretical workers.[17,18]. There is some recent calculations[19,20] which also reveals the similar facts.

Table T1

An evaluated result of excitation energy $\in(q_x)$ as a function of q_x for different values of momentum k namely (a) k=1 (b) k=2.5, (c) k=4 with parameter $C_1=10$, $C_2=4$ and $\gamma=1$

q_x	Excitation energy $\in(q_x)$		
	k=1	k=2.5	k=4
-2.0	4.5	8.7	12.5
-1.5	3.2	7.6	11.6
-1.0	2.6	6.2	10.2
-0.5	1.3	5.6	8.5
0.0	0.0	4.8	7.2
0.5	1.8	5.9	8.9
1.0	2.9	6.7	10.6
1.5	3.5	8.3	12.8
2.0	4.8	9.5	14.2

Table T2

An evaluated result of excitation energy $\in(q_y)$ as a function of q_y in the direction of y-axis for different momentum k namely (a) k=1, (b) k=2.5 and (c) k=4 with parameter C_1=10 and C_2=4 and γ =1

q_y	$\in(q_y)$		
	k=1	k=2.5	k=4
-4.0	12.3	5.6	0.84
-3.0	10.8	4.2	0.97
-2.0	7.6	3.7	1.84
-1.0	5.3	2.8	2.67
-0.5	3.2	1.5	3.08
0.0	1.6	0.6	4.16
0.5	2.8	1.9	5.7
1.0	3.3	3.8	6.8
1.5	4.2	3.6	7.9
2.0	5.7	5.4	8.4

Table T3

An evaluated result of the critical velocity v_c of BEC as a function of SOC parameter γ for $C_1>3C_2$

γ	v_c
0	2.42
1	2.48
2	2.55
3	2.58
4	2.67
5	2.73
6	2.75
7	2.78
8	2.85

Table T4

An evaluated result of the critical velocity v_c of BEC as a function of SOC parameter γ for $C_1<3C_2$

γ	v_c
0	2.23
1	2.44
2	2.63
3	2.72
4	2.76
5	2.80
6	2.82
7	2.74
8	2.67

CONCLUSION

From above theoretical investigation and analysis, we have came across the following conclusions;

(1) Spin-orbit coupling plays an essential role in the super fluidity of SOC coupled Bose Einstein condensate.
(2) Our theoretical analysis of the Bogoliubov excitation shows that it has two branches (a) one is gapless and phonon like at long wavelength (b) typically gapped.
(3) The excitation implies super fluidity which contains distinct new features: (i) Galilean invariance is absent (ii) One cannot define critical velocity of super fluidity independent of the reference frame. The super fluidity depends upon two factors (a) the speed of BEC exceeds a critical value (b) cross-helicity. The cross helicity is defined as the cross product of the spin and kinetic momentum of the BEC.

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